B Separate Appendix

B.1 Computational Algorithm

We use a finite history of length n of the aggregate shocks to (reasonably) accurately compute the equilibrium. The variable n determines the set of aggregate finite histories $S(n)$ that we are keeping track of, and $s \in S(n)$ denotes a generic member. The number of elements of $S(n)$ is given by $n^{\#Z}$, where $\#Z$ is the number of aggregate states. The individual state is then given by his multiplier, the finite aggregate history, and his individual shock; besides his multiplier, there are $n^{\#Z} * \#N$ states for the individual.

The algorithm works as follows. Assume that we have a matrix $g(s, s')$, which gives the value of our moment $h(z^{t+1})/h(z^t)$ in the case where the transition is from finite history s to finite history s'. Given this matrix we can compute the aggregate state price in the stationary version of the economy, which we will denote by $\hat{P}(s')/\hat{P}(s)$. In computing the equilibrium, we find it more convenient to keep track of agents by their consumption share c rather than their (normalized) multiplier ζ. Note that $c^{-\alpha} = \zeta$.

To compute $\hat{D}(c, s, \eta)$, we first assume that c is unchanged and we simply use the promised savings equation to compute $\hat{D}_0$. Then, to compute $\hat{D}_{j+1}$ given $\hat{D}_j$ we do the following algorithm:

1. We start with a savings grid where the highest savings level is the debt/savings limit. Note that since this is a fraction of the net present value of income, we can compute this directly given g.

2. For savings grid point S_i, we can compute the associated consumption shares $c'(s', \eta')$, where $S_i = \hat{D}_j(c'(s', \eta'), s', \eta')$. Since $\hat{D}_j$ is piecewise linear, it is trivial to invert this function.

3. Given S_i and $\tilde{c}'(s', \eta')$, we can compute the consumption share today from the optimality condition for state today (s, η). This is given by

$$E\left\{\tilde{c}'(s', \eta')^{-\alpha} | s, \eta\right\} g(s, s')^{\alpha} = c(s, \eta)^{-\alpha},$$

   If we do this for every grid point savings grid tomorrow, fixing the state today (s, η), this yields a vector of current consumption shares $\mathbf{c}$ and their future associated net savings levels $\mathbf{S}'$ for each possible transition (η, s, s'). We can then fit linear piecewise linear functions to the $[\mathbf{c}, \mathbf{S}']$ for each transition (η, s, s'). Hence we have constructed $S'(c; \eta, s, s')$.

4. Given these piecewise linear functions $S'(c; \eta, s, s')$, we can compute trivially compute $\hat{D}_{j+1}(c, s, \eta)$ for each current consumption share c in our grid by our recursive saving equation since c is the consumption share today and we have already computed the future savings levels via our piecewise linear function for each possible future transition (η, s, s'). In this way, we can compute a vector of current consumption shares $\mathbf{c}$ and their associated current net savings levels $\hat{\mathbf{D}}_{j+1}$. We can then fit linear piecewise linear functions to the $\left[\mathbf{c}, \hat{\mathbf{D}}_{j+1}\right]$ for each (s, η). In so doing we have constructed the function $\hat{D}_{j+1}(c, s, \eta)$.

5. The iterations continues until the $\hat{D}_j$ functions converge. As one of the products of this computation we have the vectors $\mathbf{c}$ and $\mathbf{c}'(\eta')$ for each transition (η, s, s'). We store these vectors in an array and use them in our simulation step when we update the values of $g(s, s')$ implied by our transition functions for consumption shares.

6. To simulate our economy and update H, we take a single panel draw of aggregate and idiosyncratic shocks. We then compute the updated consumption shares, where each period we normalize the consumption shares to average 1, and use the normalization factor to generate a revised estimate $g'(s, s')$. Given this revised estimate we repeat the iterations until the estimate of H' converges.

B.2 Properties of Aggregate Multiplier

Corrolary B.1. *Fix a cumulative multiplier ζ. In the absence of binding solvency constraints, a z-complete trader consumes less on average in the next period than a complete trader, strictly less if marginal utility is strictly convex.*

Proof. Proof of Corollary B.1: In fact, Corollary (3.3) and (3.2) imply that if we take two traders with the same initial ζ, the complete trader will always choose higher average consumption than the z-complete trader, irrespective of $\{h\}$:

$$E\{\zeta^{-\alpha}(z^{t+1},\eta^{t+1})|z^{t+1}\} < E\{\zeta(z^{t+1},\eta^{t+1})|z^{t+1}\}^{-\alpha} = \zeta(z^t,\eta^t)^{-\alpha}.$$

□

Proposition B.1. *Suppose there are only complete or z-complete traders. The equilibrium stochastic process $\{h_t(z^t)\}$ is non-decreasing:*

$$\left(\frac{h(z^{t+1})}{h(z^t)}\right) \geq 1$$

Proof of Proposition B.1:

Proof. If the solvency constraints do not bind anywhere, then we know that on average

$$\sum_{\eta'} \zeta\left(z^{t+1},\eta^{t+1}\right)\pi(\eta^{t+1},z^{t+1}|\eta^t,z^t) = \zeta(z^t,\eta^t),$$

from equation (3.11). In that case, $h(z^{t+1}) = h(z^t)$ for all z^t. This implies that

$$\frac{h(z^{t+1})}{h(z^t)} > 1 \text{ for all } z^t$$

To see why, note that

$$h(z^{t+1}) - h(z^t)$$

$$= \int \sum_{\eta^{t+1}\succ\eta^t} \left\{ \left[T^z(z^{t+1},\eta^{t+1}|z^t,\eta^t)(\zeta(z^t,\eta^t))\right]^{\frac{-1}{\alpha}} \frac{\pi(\eta^{t+1},z^{t+1}|\eta^t,z^t)}{\pi(z^{t+1}|z^t)} - \zeta(z^t,\eta^t)^{\frac{-1}{\alpha}} \right\} d\Phi$$

Now, we know that

$$\sum_{\eta^{t+1}\succ\eta^t} \frac{\pi(\eta^{t+1},z^{t+1}|\eta^t,z^t)}{\pi(z^{t+1}|z^t)} \left[T^z(z^{t+1},\eta^{t+1}|z^t,\eta^t)(\zeta(z^t,\eta^t))\right] \leq \zeta(z^t,\eta^t),$$

with strict inequality if the debt bounds bind. From Jensen's inequality, since this a strictly convex function, this implies the following inequality

$$\sum_{\eta^{t+1}\succ\eta^t} \frac{\pi(\eta^{t+1},z^{t+1}|\eta^t,z^t)}{\pi(z^{t+1}|z^t)} \left[T^z(z^{t+1},\eta^{t+1}|z^t,\eta^t)(\zeta(z^t,\eta^t))\right]^{-\frac{1}{\alpha}}$$

$$> \left[\sum_{\eta^{t+1}\succ\eta^t} \frac{\pi(\eta^{t+1},z^{t+1}|\eta^t,z^t)}{\pi(z^{t+1}|z^t)} \left[T^z(z^{t+1},\eta^{t+1}|z^t,\eta^t)(\zeta(z^t,\eta^t))\right]\right]^{-\frac{1}{\alpha}}$$

$$\geq \left[\zeta(z^t,\eta^t)\right]^{-\frac{1}{\alpha}},$$

with strict inequality if the debt bounds bind. This implies that, piece-by-piece, the elements in the integrand are non-negative, which implies that $h(z^{t+1}) - h(z^t) > 0$. □

Proof of Proposition B.1:

Proof. If the solvency constraints do not bind anywhere, then we know that on average

$$\zeta\left(z^{t+1},\eta^{t+1}\right)\pi(\eta^{t+1},z^{t+1}|\eta^t,z^t) = \zeta(z^t,\eta^t),$$

from equation (3.11). In that case, $h(z^{t+1}) = h(z^t)$ for all z^t. This implies that

$$\frac{h(z^{t+1})}{h(z^t)} > 1 \text{ for all } z^t$$

To see why, note that

$$h(z^{t+1}) - h(z^t)$$

$$= \int \sum_{\eta^{t+1}\succ\eta^t} \left\{\left[T^{com}(z^{t+1},\eta^{t+1}|z^t,\eta^t)(\zeta(z^t,\eta^t))\right]^{\frac{-1}{\alpha}} \frac{\pi(\eta^{t+1},z^{t+1}|\eta^t,z^t)}{\pi(z^{t+1}|z^t)} - \zeta(z^t,\eta^t)^{\frac{-1}{\alpha}}\right\} d\Phi$$

Now, we know that

$$\left[T^{com}(z^{t+1},\eta^{t+1}|z^t,\eta^t)(\zeta(z^t,\eta^t))\right] \leq \zeta(z^t,\eta^t),$$

with strict inequality if the debt bounds bind. From Jensen's inequality, since this a strictly convex function, this implies the following inequality

$$\left[T^{com}(z^{t+1},\eta^{t+1}|z^t,\eta^t)(\zeta(z^t,\eta^t))\right]^{-\frac{1}{\alpha}}$$

$$\geq \left[\zeta(z^t,\eta^t)\right]^{-\frac{1}{\alpha}},$$

with strict inequality if the debt bounds bind. This implies that, piece-by-piece, the elements in the integrand are non-negative, which implies that $h(z^{t+1}) - h(z^t) > 0$. □

Proposition B.2. *Suppose there are only diversified investors. In the case of independence, the equilibrium stochastic process $\{h_t(z^t)\}$ is non-decreasing on average under the risk-neutral measure:*

$$\sum_{z^{t+1}\succ z^t} \widetilde{\phi}(z^{t+1}|z^t)\left(\frac{h(z^{t+1})}{h(z^t)}\right) \geq 1$$

Proof of Proposition B.2:

Proof. Note that:

$$h(z^{t+1}) - h(z^t)$$

$$= \int \sum_{\eta^{t+1} \succ \eta^t} \left\{ \left[T^{eq}(z^{t+1}, \eta^{t+1} | z^t, \eta^t)(\zeta(z^t, \eta^t)) \right]^{\frac{-1}{\alpha}} \varphi(\eta^{t+1} | \eta^t) - \zeta(z^t, \eta^t)^{\frac{-1}{\alpha}} \right\} d\Phi$$

Now, we know that

$$\sum_{z^{t+1} \succ z^t} \widetilde{\phi}(z^{t+1} | z^t) \sum_{\eta^{t+1} \succ \eta^t} \varphi(\eta^{t+1} | \eta^t) \left[T^{eq}(z^{t+1}, \eta^{t+1} | z^t, \eta^t)(\zeta(z^t, \eta^t)) \right] \leq \zeta(z^t, \eta^t),$$

with strict inequality if the debt bounds bind. From Jensen's inequality, since this a strictly convex function, this implies the following inequality

$$\sum_{z^{t+1} \succ z^t} \widetilde{\phi}(z^{t+1} | z^t) \sum_{\eta^{t+1} \succ \eta^t} \varphi(\eta^{t+1} | \eta^t) \left[T^{eq}(z^{t+1}, \eta^{t+1} | z^t, \eta^t)(\zeta(z^t, \eta^t)) \right]^{-\frac{1}{\alpha}}$$

$$> \left[\sum_{z^{t+1} \succ z^t} \widetilde{\phi}(z^{t+1} | z^t) \sum_{\eta^{t+1} \succ \eta^t} \varphi(\eta^{t+1} | \eta^t) \left[T^{eq}(z^{t+1}, \eta^{t+1} | z^t, \eta^t)(\zeta(z^t, \eta^t)) \right] \right]^{-\frac{1}{\alpha}}$$

$$\geq \left[\zeta(z^t, \eta^t) \right]^{-\frac{1}{\alpha}},$$

with strict inequality if the debt bounds bind. This implies that, piece-by-piece, the elements in the integrand are non-negative, which implies that $\sum_{z^{t+1} \succ z^t} \widetilde{\phi}(z^{t+1} | z^t) h(z^{t+1}) - h(z^t) > 0$. ☐

B.3 Ex Ante Heterogeneity

Suppose there are some differences in permanent income and initial endowments of financial wealth. Let x_y index the permanent component, meaning that a household with label x_y receives x_y times the labor income process and the initial endowment of financial wealth of the average household. The only part that affects the stationary equilibrium is the labor income part.

Lemma B.1. *If the borrowing constraints are proportional to x_y, then optimal consumption is proportional to x_y as well.*

This lemma implies that the fraction μ_i can be interpreted as the fraction of human wealth (not financial wealth) held by households in segment i. For example, if μ_z is calibrated to 5%, that really means 5% of human wealth is held by z-complete traders (not 5% of the population).

Proof of Lemma B.1:

Proof. We use $\widetilde{P}(z^t, \eta^t)$ to denote $P(z^t)\pi(z^t, \eta^t)$. Let γ denote the multiplier on the present-value budget constraint, let $\nu(z^t, \eta^t)$ denote the multiplier on the measurability constraint in node (z^t, η^t), and, finally, let $\varphi(z^t, \eta^t)$ denote the multiplier on the debt constraint. We consider the case in which the borrowing constraint is $x_y \underline{M}_t$ – proportional in x_y. We consider the case in which the initial endowment of the diversifiable income claim is proportional to x_y as well. Let $\{c, \hat{a}\}$ denote the optimal consumption and asset choices for a household with $x_y = 1$. Using the proportionality assumptions and the consumption conjecture, the saddle point problem for a household with permanent income x_y can be stated as:

$$\begin{aligned}
L(x_y) = \quad & \sum_{t=1}^{\infty} \beta^t \sum_{(z^t,\eta^t)} u(c(z^t,\eta^t))\pi(z^t,\eta^t) x_y^{1-\alpha} \\
& + x_y \widehat{\gamma}_0 \left\{ \sum_{t \geq 1} \sum_{(z^t,\eta^t)} \widetilde{P}(z^t,\eta^t) \left[\gamma Y(z^t)\eta_t - c(z^t,\eta^t) \right] + \varpi(z^0) \right\} \\
& + x_y \sum_{t \geq 1} \sum_{z^t,\eta^t} \widehat{\nu}(z^t,\eta^t) \\
& \left\{ \sum_{\tau \geq t} \sum_{(z^\tau,\eta^\tau) \succ (z^t,\eta^t)} \widetilde{P}(z^\tau,\eta^\tau) \left[\gamma Y(z^\tau)\eta_\tau - c(z^\tau,\eta^\tau) \right] - \widetilde{P}(z^t,\eta^t) \hat{a}(z^t,\eta^{t-1}) \right\} \\
& + x_y \sum_{t \geq 1} \sum_{(z^t,\eta^t)} \widehat{\varphi}(z^t,\eta^t) \\
& \left\{ \underline{M}_t(z^t,\eta^t) \widetilde{P}(z^t,\eta^t) - \sum_{\tau \geq t} \sum_{(z^\tau,\eta^\tau) \succ (z^t,\eta^t)} \widetilde{P}(z^\tau,\eta^\tau) \left[\gamma Y(z^\tau)\eta_\tau - c(z^\tau,\eta^\tau) \right] \right\}.
\end{aligned}$$

Let $\{\gamma, \nu, \varphi\}$ denote the saddle point multipliers for a household with $x_y = 1$. Then it is easy to see that $\{\widehat{\gamma}, \widehat{\nu}, \widehat{\varphi}\} = x_y^{-\alpha}\{\gamma, \nu, \varphi\}$ and $x_y\{c, \hat{a}\}$ is a saddle point as well.

□

B.4 Other preferences

Our analytic framework extends readily to the case of Epstein and Zin (1989)'s recursive preferences since these preferences also feature the homogeneity of the inverse of marginal utility over consumption. To show this, assume that preferences are defined by the following recursion:

$$V_t = \left[(1-\beta) c_t^{1-\rho} + \beta (\mathcal{R}_t V_{t+1})^{1-\rho} \right]^{1/(1-\rho)},$$

where $\mathcal{R}$ is a twisted expectations operator:

$$\mathcal{R}_t V_{t+1} = \left(E_t \left[V_{t+1}^{1-\gamma} \right] \right)^{1/(1-\gamma)}.$$

We define the following adjusted cumulative multiplier:

$$\widetilde{\zeta}(z^t,\eta^t) = \frac{\zeta(z^t,\eta^t)}{M^t(z^t,\eta^t)}.$$

and the $-1/\rho$-th moment of these weights:

$$h(z^t) = \sum_{\eta^t} \widetilde{\zeta}(z^t,\eta^t)^{-\frac{1}{\rho}} \pi(\eta^t|z^t).$$

Proposition B.3. *In the case of Epstein-Zin preferences, the trader's consumption satisfies the following* *rule:*

$$c(z^t,\eta^t) = \frac{\widetilde{\zeta}(z^t,\eta^t)^{-\frac{1}{\rho}}}{h(z^t)} C(z^t) \tag{B.1}$$

and the pricing kernel is given by:

$$\frac{P(z^{t+1})}{P(z^t)} = \beta \left(\frac{C(z^{t+1})}{C(z^t)}\right)^{-\rho} \left(\frac{h(z^{t+1})}{h(z^t)}\right)^{\rho},$$

The consumption sharing rule and the main aggregation result go through in the case of recursive preferences.

Proof of Proposition B.3:

Proof. This change in preferences would change the first-order condition with respect to consumption $c(z^t,\eta^t)$ (3.8) (which is common to all our asset structures) to

$$\frac{\partial V_0}{\partial c_t} = \zeta(z^t,\eta^t)P(z^t)\pi(z^t,\eta^t) \tag{B.2}$$

where $\zeta(z^t,\eta^t)$ satisfies our multiplier recursion (3.7).

To derive an expression for $\partial V_0/\partial c_t$, note first that

$$\frac{\partial V(z^t,\eta^t)}{\partial c(z^t,\eta^t)} = V(z^t,\eta^t)^{\rho}(1-\beta)c(z^t,\eta^t)^{-\rho},$$

and

$$\frac{\partial V(z^t,\eta^t)}{\partial c(z^{t+1},\eta^{t+1})} = \beta \frac{\partial V(z^t,\eta^t)}{\partial c(z^t,\eta^t)} \left[\frac{V(z^{t+1},\eta^{t+1})}{E_t\left(V_{t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}\right]^{\rho-\gamma} \left[\frac{c(z^{t+1},\eta^{t+1})}{c(z^t,\eta^t)}\right]^{-\rho} \pi(z^{t+1},\eta^{t+1} \rvert z^t,\eta^t).$$

Using the chain rule, these expression imply that

$$\frac{\partial V(z^{t-1},\eta^{t-1})}{\partial c(z^{t+1},\eta^{t+1})} = \left(\frac{V(z^{t-1},\eta^{t-1})}{c(z^{t+1},\eta^{t+1})}\right)^{\rho} \beta^2(1-\beta)M(z^t,\eta^t)M(z^{t+1},\eta^{t+1})\pi(z^{t+1},\eta^{t+1}|z^{t-1},\eta^{t-1}),$$

where

$$M(z^t,\eta^t) = \left[\frac{V_t(z^t,\eta^t)}{\mathcal{R}_{t-1}V_t(z^t,\eta^t)}\right]^{\rho-\gamma}.$$

By backward induction we get that

$$\frac{\partial V_0}{\partial c(z^t,\eta^t)} = V_0^{\rho}\beta^t(1-\beta)M^t(z^t,\eta^t)c(z^t,\eta^t)^{-\rho}\pi(z^t,\eta^t), \tag{B.3}$$

where

$$M^t(z^t,\eta^t) = \Pi_{\tau=0}^{t} M(z^{\tau},\eta^{\tau}).$$

These results imply that our first-order condition (B.2) can be expressed as

$$V_0^{\rho}\beta^t(1-\beta)M^t(z^t,\eta^t)c(z^t,\eta^t)^{-\rho} = \zeta(z^t,\eta^t)P(z^t).$$

To derive the new expression for the household consumption share which replaces (A.3), note that our first-condition implies that

$$\frac{M^t(z^t,\widetilde{\eta}^t)}{M^t(z^t,\eta^t)}\frac{c(z^t,\widetilde{\eta}^t)^{-\rho}}{c(z^t,\eta^t)^{-\rho}} = \frac{\zeta(z^t,\widetilde{\eta}^t)}{\zeta(z^t,\eta^t)}.$$

This in turn implies our new consumption rule

$$c(z^t,\eta^t) = \frac{\widetilde{\zeta}(z^t,\eta^t)^{-\frac{1}{\rho}}}{h(z^t)} C(z^t) \tag{B.4}$$

where

$$\widetilde{\zeta}(z^t,\eta^t) = \frac{\zeta(z^t,\eta^t)}{M^t(z^t,\eta^t)}.$$

and where

$$h(z^t) = \sum_{\eta^t} \widetilde{\zeta}(z^t,\eta^t)^{-\frac{1}{\rho}} \pi(\eta^t|z^t).$$

To see how this changes the pricing kernel, note that our first-order condition (B.3) implies that

$$\begin{aligned}
\frac{P(z^{t+1})}{P(z^t)} &= \beta \frac{\widetilde{\zeta}(z^t,\eta^t)}{\widetilde{\zeta}(z^{t+1},\eta^{t+1})}\left[\frac{c(z^{t+1},\eta^{t+1})}{c(z^t,\eta^t)}\right]^{-\rho} \\
&= \beta \frac{\widetilde{\zeta}(z^t,\eta^t)}{\widetilde{\zeta}(z^{t+1},\eta^{t+1})}\left[\frac{\widetilde{\zeta}(z^{t+1},\eta^{t+1})^{-\frac{1}{\rho}}}{\widetilde{\zeta}(z^t,\eta^t)^{-\frac{1}{\rho}}}\frac{C(z^{t+1})}{C(z^t)}\frac{h(z^t)}{h(z^{t+1})}\right]^{-\rho} \\
&= \beta \left(\frac{C(z^{t+1})}{C(z^t)}\right)^{-\rho}\left(\frac{h(z^{t+1})}{h(z^t)}\right)^{\rho},
\end{aligned}$$

where we use (B.4). □

B.5 Portfolio Choice

Consider the problem of an agent who is choosing how much to hold of two assets which offer returns $R_1(z)$ and $R_2(z)$. The value of his total portfolio next period will be given by

$$\hat{a}(z) = x_1 R_1(z) + x_2 R_2(z),$$

where x_1 and x_2 denote the amounts invested in the respective assets. This implies a certain relationship between the set of possible wealth realizations that he can have tomorrow:

$$\frac{\hat{a}(z)}{R(z;x)} = \frac{\hat{a}(\tilde{z})}{R(\tilde{z};x)}.$$

for some x where

$$R(z;x) \equiv xR_1(z) + (1-x)R_2(z).$$

More generally, we can think of $R(z^t; x(z^{t-1},\eta^{t-1}))$ determining a vector of returns given a choice of asset weights x. Rather than look at this in terms of final payouts $\hat{a}$, a more informative way of thinking about this, is taking $\hat{b}(z^{t-1},\eta^{t-1})$ to be his savings and note that his subsequent asset position is given by

$$\hat{b}(z^{t-1},\eta^{t-1})R(z^t; x(z^{t-1}(z^t),\eta^{t-1}) = \hat{a}(z^t,\eta^t)$$

For this agent, his problem can be written as

$$L = \min_{\{\gamma,\nu,\varphi\}} \max_{\{c,\hat{b},x\}} \sum_{t=1}^{\infty} \beta^t \sum_{(z^t,\eta^t)} u(c(z^t,\eta^t))\pi(z^t,\eta^t)$$

$$+\gamma \left\{ \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \widetilde{P}(z^t,\eta^t) \left[\gamma Y(z^t)\eta_t - c(z^t,\eta^t)\right] + \varpi(z^0) \right\}$$

$$+\sum_{t\geq 1} \sum_{z^t,\eta^t} \nu(z^t,\eta^t) \left\{ \begin{array}{c} \sum_{\tau\geq t} \sum_{(z^\tau,\eta^\tau)\succ(z^t,\eta^t)} \widetilde{P}(z^\tau,\eta^\tau)\left[\gamma Y(z^\tau)\eta_\tau - c(z^\tau,\eta^\tau)\right] \\ -\widetilde{P}(z^t,\eta^t)\hat{b}(z^{t-1},\eta^{t-1})R(z^t;x(z^{t-1},\eta^{t-1})) \end{array} \right\}$$

$$+\sum_{t\geq 1} \sum_{(z^t,\eta^t)} \varphi(z^t,\eta^t) \left\{ \underline{M}_t(z^t,\eta^t)\widetilde{P}(z^t,\eta^t) - \sum_{\tau\geq t} \sum_{(z^\tau,\eta^\tau)\succ(z^t,\eta^t)} \widetilde{P}(z^\tau,\eta^\tau)\left[\gamma Y(z^\tau)\eta_\tau - c(z^\tau,\eta^\tau)\right] \right\}$$

This implies our standard set of condition in terms of the recursive multipliers

$$\zeta(z^t,\eta^t) = \zeta(z^{t-1},\eta^{t-1}) + \nu\left(z^t,\eta^t\right) - \varphi(z^t,\eta^t), \tag{B.5}$$

:

$$\beta^t u'(c(z^t,\eta^t))\pi(z^t,\eta^t) - \zeta(z^t,\eta^t)P(z^t)\pi(z^t,\eta^t) = 0 \tag{B.6}$$

and our standard passive trader Martingale condition

$$\sum_{\substack{z^{t+1}\succ z^t \\ \eta^{t+1}\succ\eta^t}} \left[\nu\left(z^{t+1},\eta^{t+1}\right) R(z^t;x(z^{t-1}(z^t),\eta^{t-1})\right] \pi(z^{t+1},\eta^{t+1})P(z^{t+1}) = 0, \tag{B.7}$$

along with the additional condition given by

$$-\sum_{z^t,\eta^t} \nu(z^t,\eta^t)\widetilde{P}(z^t,\eta^t)\frac{\partial R(z^t;x(z^{t-1},\eta^{t-1}))}{\partial x(z^{t-1},\eta^{t-1})} = 0.$$

It's an orthogonality condition on the marginal returns weighted by the shadow price of the measurability condition. We can rewrite this condition in terms of our cumulative multipliers as

$$-\sum_{z^t,\eta^t} \left[\zeta(z^t,\eta^t) - \zeta(z^{t-1},\eta^{t-1}) + \varphi(z^t,\eta^t)\right] \widetilde{P}(z^t,\eta^t)\frac{\partial R(z^t;x(z^{t-1},\eta^{t-1}))}{\partial x(z^{t-1},\eta^{t-1})} = 0.$$

B.6 Asset Pricing in the MP economy

The asset pricing moments for the MP economy are reported in Table 11. In the RA version of the MP economy (column 1), the risk-free and the conditional market price of risk are no longer constant. However, the heterogeneity in trading technologies increases risk premia, creates more time-variation in risk premia without increasing the volatility of the risk-free rate. The maximum Sharpe ratio is .48, while the Sharpe ratio on equity in post-war US data is .45. The conditional market price of risk has a standard deviation of 5.4%. The model produces a low risk-free rate of .86 %, a large risk premium on equity of 5.8 %. The risk-free rate in the MP economy is more volatile (2.8 %), but not more so than the RA risk-free rate (3%). The standard deviation of the conditional market price of risk is 5.4 %, compared to 1.1 % in the data. The additional risk-free rate variation brings the volatility of equity returns in line with the data. In addition, the MP economy comes close to matching the autocorrelation properties of the returns we observe in the data. The autocorrelation is -.19 in the model, as in the data. The contemporaneous correlation of returns on equity and the risk-free rate is .2 in the model, compared to .27 in the data.

[Table 11 about here.]

Table 11: Asset Pricing in the MP Economy

	RA	**HTT**	**Data**
$E[R_f]$	13.04	0.866	1.049
$\sigma[R_f]$	3.144	2.897	1.560
$\sigma[m]/E[m]$	0.193	0.481	
$Std[\sigma_t[m]/E_t[m]]$	0.011	0.054	
$E[R_{eq} - R_f]$	2.324	5.861	7.531
$\sigma[R_{eq} - R_f]$	13.34	12.49	16.94
$E[R_{eq} - R_f]/\sigma[R_{eq} - R_f]$	0.174	0.469	0.444
$E[R_{lc} - R_f]$	4.397	10.87	7.531
$\sigma[R_{lc} - R_f]$	23.07	22.87	16.94
$E[R_{lc} - R_f]/\sigma[R_{lc} - R_f]$	0.190	0.475	0.444
$E[PD]_{eq}$	7.989	18.72	33.87
$\sigma[PD]_{eq}$	12.81	15.20	16.78
$\rho[R_{eq}, R_f]$	0.204	0.204	0.272
$\rho[R_{eq}(t), R_{eq}(t-1)]$	-0.193	-0.199	-0.191
$\rho[R_{lc}(t), R_{lc}(t-1)]$	-0.103	-0.134	-0.191
$E[R_b - R_f]$	0.449	-0.604	1.070
$\sigma[R_b - R_f]$	2.337	1.297	9.366
$[E(R_b - R_f)]/[\sigma(R_b - R_f)]$	0.192	-0.466	0.114

Notes: Parameters setting: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 0.1. The simulation moments are generated by 10000 draws from an economy with 3000 agents. Benchmark calibration of idiosyncratic shocks and MP calibration of aggregate shocks. Reports the moments of asset prices for the RA (Representative Agent) economy, for the HTT (Heterogeneous Trading Technology) economy and for the data. We use post-war US annual data for 1946-2005. The market return is the CRSP value weighted return for NYSE/NASDAQ/AMEX. We use the Fama risk-free rate series from CRSP (average 3-month yield). To compute the standard deviation of the risk-free rate, we compute the annualized standard deviation of the ex post real monthly risk-free rate. The return on the long-run bond is measured using the Bond Total return index for 30-year US bonds from Global Financial Data.