Technical Appendix for "Involuntary Unemployment and the Business Cycle"

by

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Apart from sections B and D below, the derivations in this appendix are standard and can be found in the technical appendices in ACEL and CEE, for example. We display them here so that the technical results are all available in one place and in a consistent notation. The material in sections B and D involve straightforward (but, sometimes tedious) extensions of the results in the manuscript. Equation numbers refer to equation numbers in the text of the paper.

A. Equilibrium Conditions in the Model Without Capital

We first derive the equilibrium conditions associated with price setting. We then derive the other conditions.

A.1. Price Setting Equilibrium Conditions

The discounted profits of the

$$E_t \sum_{j=0}^{\infty} \beta^j \ \upsilon_{t+j} \overbrace{\left[\overbrace{P_{i,t+j} Y_{i,t+j}}^{\text{revenues}} - \overbrace{P_{t+j} s_{t+j} Y_{i,t+j}}^{\text{total cost}} \right]}^{\text{period } t+j \text{ profits sent to household}},$$

where υ_{t+j} denotes the period $t+j$ Lagrange multiplier on household budget constraint. Let $\tilde{P}_t$ denote the price selected by each of the $1-\xi_p$ firms that have opportunity to reoptimize price in period t. Because firms have no state variables, they are only concerned about future histories in which they cannot reoptimize price. This leads to the following objective function for a firm that can reoptimize price in period t :

$$E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \upsilon_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right].$$

Substitute out for intermediate good firm output using the demand curve:

$$E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \upsilon_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[\tilde{P}_t^{1-\varepsilon} - P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon} \right],$$

where

$$\varepsilon \equiv \frac{\lambda_f}{\lambda_f - 1}$$

Differentiate with respect to $\tilde{P}_t$:

$$E_t \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j \upsilon_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[(1-\varepsilon) \left(\tilde{P}_t\right)^{-\varepsilon} + \varepsilon P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon-1} \right] = 0,$$

or,

$$E_t \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j \upsilon_{t+j} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \lambda_f s_{t+j} \right] = 0.$$

Note that when $\xi_p = 0$, one obtains the standard result, that price is fixed markup over marginal cost.

Now, substitute out the multiplier:

$$E_t \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j \overbrace{\frac{u'\left(C_{t+j}\right)}{P_{t+j}}}^{\text{marginal utility of currency } = \upsilon_{t+j}} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \lambda_f s_{t+j} \right] = 0,$$

or, given our assumption about log utility,

$$E_t \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j \frac{Y_{t+j}}{C_{t+j}} P_{t+j}^{\varepsilon} \left[\frac{\tilde{P}_t}{P_{t+j}} - \lambda_f s_{t+j} \right] = 0.$$

or,

$$E_t \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \left[\tilde{p}_t X_{t,j} - \lambda_f s_{t+j}\right] = 0,$$

where

$$\tilde{p}_t = \frac{\tilde{P}_t}{P_t}, X_{t,j} = \begin{cases} \frac{1}{\pi_{t+j}\pi_{t+j-1}\cdots\pi_{t+1}}, & j \geq 1 \\ 1, & j = 0. \end{cases}, X_{t,j} = X_{t+1,j-1}\frac{1}{\pi_{t+1}}, j > 0$$

Solving for $\tilde{p}_t$:

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \lambda_f s_{t+j}}{E_t \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{1-\varepsilon}} = \frac{K_{p,t}}{F_{p,t}},$$

We now obtain simple recursive expressions for $K_{p,t}$ and $F_{p,t}$.

Consider $K_{p,t}$ first. Accordingly,

$$\begin{aligned}
K_{p,t} &= E_t \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\frac{\lambda_f}{\lambda_f-1}} \lambda_f s_{t+j} \\
&= \lambda_f \frac{Y_t}{C_t} s_t + \beta\xi_p E_t \sum_{j=1}^{\infty} \left(\beta\xi_p\right)^{j-1} \frac{Y_{t+j}}{C_{t+j}} \left(\frac{1}{\pi_{t+1}} X_{t+1,j-1}\right)^{-\frac{\lambda_f}{\lambda_f-1}} \lambda_f s_{t+j} \\
&= \lambda_f \frac{Y_t}{C_t} s_t + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{-\frac{\lambda_f}{\lambda_f-1}} \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j X_{t+1,j}^{-\frac{\lambda_f}{\lambda_f-1}} \lambda_f \frac{Y_{t+1+j}}{C_{t+1+j}} s_{t+1+j} \\
&= \lambda_f \frac{Y_t}{C_t} s_t + \beta\xi_p \overbrace{E_t E_{t+1}}^{=E_t} \left(\frac{1}{\pi_{t+1}}\right)^{-\frac{\lambda_f}{\lambda_f-1}} \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j X_{t+1,j}^{-\frac{\lambda_f}{\lambda_f-1}} \lambda_f \frac{Y_{t+1+j}}{C_{t+1+j}} s_{t+1+j} \\
&= \lambda_f \frac{Y_t}{C_t} s_t + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{-\frac{\lambda_f}{\lambda_f-1}} E_{t+1} \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j X_{t+1,j}^{-\frac{\lambda_f}{\lambda_f-1}} \lambda_f \frac{Y_{t+1+j}}{C_{t+1+j}} s_{t+1+j} \\
&= \lambda_f \frac{Y_t}{C_t} s_t + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{-\frac{\lambda_f}{\lambda_f-1}} K_{p,t+1}
\end{aligned}$$

so that

$$K_{p,t} = \lambda_f \frac{Y_t}{C_t} s_t + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} K_{p,t+1}. \tag{A.1}$$

Similarly,

$$F_{p,t} \equiv E_t \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{\frac{\lambda_f}{1-\lambda_f}} = \frac{Y_t}{C_t} + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} F_{p,t+1} \tag{A.2}$$

In (A.1), marginal cost is defined in (2.28) and (2.29), repeated here for convenience:

$$s_t = \frac{1}{\lambda_f} \frac{C_t z_h\left(h_t, \varsigma_t\right)}{A_t}.$$

Evaluating (2.27)

$$P_t = \left[\left(1-\xi_p\right) \tilde{P}_t^{\frac{1}{1-\lambda_f}} + \xi_p P_{t-1}^{\frac{1}{1-\lambda_f}}\right]^{1-\lambda_f}.$$

Dividing by P_t and rearranging, we obtain:

$$\tilde{p}_t = \left[\frac{1-\xi_p \pi_t^{\frac{1}{\lambda_f-1}}}{1-\xi_p}\right]^{1-\lambda_f} \tag{A.3}$$

We conclude that the equilibrium conditions associated with price setting are (A.1), (A.2) and:

$$\left[\frac{1-\xi_p \pi_t^{\frac{1}{\lambda_f-1}}}{1-\xi_p}\right]^{1-\lambda_f} = \frac{K_{p,t}}{F_{p,t}}. \tag{A.4}$$

In a zero inflation steady state without price distortions,

$$K_p = F_p \tag{A.5}$$

according to (A.4). It then follows from (A.1) and (A.2) that

$$s = 1/\lambda_f,\ F_p = \frac{\frac{1}{1-\eta_g}}{1-\beta\xi_p}, \tag{A.6}$$

where $1/\left(1-\eta_g\right)$ is the steady state output to consumption ratio, Y_t/C_t, and η_g is the steady state G_t/Y_t ratio. the object, η_g, is something we fix in the calculations. See below for further discussion.

In the analysis of the linearized equilibrium conditions in the manuscript, we set $g_t, \eta_g \equiv 0$. Differentiating (A.1), (A.2) and (A.4) in steady state:

$$\begin{aligned}
\hat{K}_{p,t} &= \left(1-\beta\xi_p\right)\hat{s}_t + \beta\xi_p\left[\frac{\lambda_f}{\lambda_f-1}\hat{\pi}_{t+1} + \hat{K}_{p,t+1}\right] \\
\hat{F}_{p,t} &= \beta\xi_p\left[\frac{1}{\lambda_f-1}\hat{\pi}_{t+1} + \hat{F}_{p,t+1}\right] \\
\hat{K}_{p,t} &= \frac{\xi_p}{1-\xi_p}\hat{\pi}_t + \hat{F}_{p,t}
\end{aligned}$$

Substitute the third equation into the first:

$$\frac{\xi_p}{1-\xi_p}\hat{\pi}_t + \hat{F}_{p,t} = \left(1-\beta\xi_p\right)\hat{s}_t + \beta\xi_p\left[\frac{\lambda_f}{\lambda_f-1}\hat{\pi}_{t+1} + \frac{\xi_p}{1-\xi_p}\hat{\pi}_{t+1} + \hat{F}_{p,t+1}\right]$$

Substitute the recursive expression for $\hat{F}_{p,t}$:

$$\frac{\xi_p}{1-\xi_p}\hat{\pi}_t + \beta\xi_p\left[\frac{1}{\lambda_f-1}\hat{\pi}_{t+1} + \hat{F}_{p,t+1}\right] = \left(1-\beta\xi_p\right)\hat{s}_t + \beta\xi_p\left[\frac{\lambda_f}{\lambda_f-1}\hat{\pi}_{t+1} + \frac{\xi_p}{1-\xi_p}\hat{\pi}_{t+1} + \hat{F}_{p,t+1}\right],$$

and rearrange, to obtain:

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p}\hat{s}_t.$$

This is the linearized Phillips curve used in the text.

A.2. Other Private Sector Equilibrium Conditions

We now derive the equilibrium relationship between aggregate consumption and aggregate inputs, (2.35), using the approach described in Yun (1996). Let Y_t^* denote the unweighted integral of intermediate inputs:

$$Y_t^* \equiv \int_0^1 Y_{i,t}di = \int_0^1 A_t h_{i,t}di = A_t h_t.$$

Using the demand curve, (2.26),

$$Y_t^* = \int_0^1 Y_{i,t} di = Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{-\frac{\lambda_f}{\lambda_f - 1}} di = Y_t P_t^{\frac{\lambda_f}{\lambda_f-1}} \int_0^1 P_{i,t}^{-\frac{\lambda_f}{\lambda_f-1}} di = Y_t P_t^{\frac{\lambda_f}{\lambda_f-1}} \left(P_t^* \right)^{-\frac{\lambda_f}{\lambda_f-1}},$$

say, where

$$P_t^* \equiv \left[\int_0^1 P_{i,t}^{-\frac{\lambda_f}{\lambda_f-1}} di \right]^{\frac{-(\lambda_f-1)}{\lambda_f}} = \left[\left(1-\xi_p\right) \tilde{P}_t^{-\frac{\lambda_f}{\lambda_f-1}} + \xi_p \left(P_{t-1}^* \right)^{-\frac{\lambda_f}{\lambda_f-1}} \right]^{\frac{-(\lambda_f-1)}{\lambda_f}}.$$

Combining the preceding three equations, we obtain (2.35), which we reproduce here for convenience:

$$\begin{aligned} \frac{C_t}{A_t} &= p_t^* h_t - g_t n_t \\ \log \frac{C_t}{C_{t-1}} &= \log \left(p_t^* h_t - g_t n_t \right) - \log \left(p_{t-1}^* h_{t-1} - g_{t-1} n_{t-1} \right) + g_{A,t} \end{aligned}$$

$$G_t + C_t = p_t^* A_t h_t. \tag{A.7}$$

In (A.7), include government consumption expenditures, which we model as follows:

$$G_t = g_t N_t,$$

where $\log g_t$ is potentially a stationary stochastic process independent of any other shocks in the system, such as A_t. Also,

$$N_t = A_t^{\gamma} N_{t-1}^{1-\gamma},\ 0 < \gamma \leq 1. \tag{A.8}$$

In the extreme case, $\gamma = 1$, this reduces to the model of G_t studied in Christiano and Eichenbaum (1992). A problem with the latter model, however, is that it implies G_t moves immediately with shocks to A_t, an implication that seems implausible. With γ close to zero, the immediate impact of A_t on G_t is virtually nil. Yet, regardless of the value of γ, G_t/A_t converges to a constant in nonstochastic steady state. This is necessary if we are to have balanced growth in the case that A_t follows a growth path in the steady state. To see that G_t/A_t converges in steady state, note

$$n_t = \left(\frac{n_{t-1}}{g_{A,t}} \right)^{1-\gamma},\ n_t \equiv \frac{N_t}{A_t}, \tag{A.9}$$

so that the steady state value of n_t is:

$$n = \left(\frac{1}{g_A} \right)^{\frac{1-\gamma}{\gamma}}. \tag{A.10}$$

From this we conclude that

$$\frac{G_t}{A_t} = g_t n_t, \tag{A.11}$$

is constant in a steady state too. In our steady state analysis, we are interested in fixing η_g, the steady state ratio of G_t to Y_t :

$$\eta_g = \frac{G}{Y} = \frac{G/A}{Y/A} = \frac{g\left(\frac{1}{g_A}\right)^{\frac{1-\gamma}{\gamma}}}{h}.$$

In practice, we fix η_g, g_A and h at their empirically relevant values and so the above equation can be thought of as determining a value for g.

In (A.7), we have used the goods clearing condition, (2.32), and p_t^* captures the distortions to output due to the price setting frictions:

$$p_t^* \equiv \left(\frac{P_t^*}{P_t}\right)^{\frac{\lambda_f}{\lambda_f - 1}}.$$

The law of motion of the distortions is, using (A.3) and (A.4):

$$p_t^* = \left[(1-\xi_p)\left(\frac{1-\xi_p \pi_t^{\frac{1}{\lambda_f-1}}}{1-\xi_p}\right)^{\lambda_f} + \frac{\xi_p \pi_t^{\frac{\lambda_f}{\lambda_f-1}}}{p_{t-1}^*}\right]^{-1}. \tag{A.12}$$

By (A.1), we require an expression for $\lambda_f Y_t s_t / C_t$. After substituting out for C_t from (A.7) into the expression for marginal cost, (2.28), and using (2.29), we obtain:

$$\lambda_f \frac{Y_t}{C_t} s_t = \frac{1}{\lambda_f} \frac{C_t z_h\left(h_t, \varsigma_t\right)}{A_t} \lambda_f \frac{Y_t}{C_t} = \frac{Y_t z_h\left(h_t, \varsigma_t\right)}{A_t}.$$

Then,

$$\lambda_f \frac{Y_t}{C_t} s_t = p_t^* h_t z_h\left(h_t, \varsigma_t\right), \tag{A.13}$$

where z_h denotes the marginal disutility of labor and z is defined in (2.22). Equation (A.13) combines the marginal cost of intermediate good firms with the optimal employment decision by the family, (2.24). By (A.2) we require an expression for Y_t / C_t :

$$\frac{Y_t}{C_t} = \frac{p_t^* A_t h_t}{p_t^* A_t h_t - g_t n_t A_t} = \frac{p_t^* h_t}{p_t^* h_t - g_t n_t} \tag{A.14}$$

The family's intertemporal Euler equation is, using (2.23):

$$1 = \beta E_t \frac{p_t^* h_t - g_t n_t}{\left[p_{t+1}^* h_{t+1} - g_{t+1} n_{t+1}\right] g_{A,t+1}} \frac{R_t}{\pi_{t+1}}, \tag{A.15}$$

where we have used (A.7) to substitute out for C_t and C_{t+1} and

$$g_{A,t+1} \equiv \frac{A_{t+1}}{A_t}.$$

Writing the equilibrium conditions, (A.1) and (A.2), after using (A.13) and (A.14), we obtain:

$$K_{p,t} = p_t^* h_t z_h(h_t, \varsigma_t) + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} K_{p,t+1} \tag{A.16}$$

$$F_{p,t} = \frac{p_t^* h_t}{p_t^* h_t - g_t n_t} + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} F_{p,t+1} \tag{A.17}$$

The 6 private sector equilibrium conditions are (A.16), (A.17), (A.4), (A.9), (A.12), and (A.15). There are 7 endogenous variables:

$$h_t, p_t^*, R_t, \pi_t, K_{p,t}, F_{p,t}, n_t.$$

When government spending is zero, then (A.9) and n_t drop from the system.

A.3. Closing the Model

The previous subsection enumerated 6 equilibrium conditions for determining 7 endogenous variables. One way to close the model is to consider the Ramsey-optimal allocations. Another is to add a Taylor rule equation.

A.3.1. Ramsey-optimal Allocations

We set government spending to zero here. Substituting out for C_t using (A.7), and for s_t using (A.13), the Lagrangian representation of the Ramsey problem is:

$$\max_{p_t^*, h_t, R_t, \pi_t, F_{p,t}, K_{p,t}} E_0 \sum_{t=0}^{\infty} \beta^t \{ \log h_t + \log p_t^* - z(h_t, \varsigma_t)$$

$$+\lambda_{1t} \left[\frac{1}{p_t^* h_t} - E_t \frac{\beta}{p_{t+1}^* h_{t+1} g_{A,t+1}} \frac{R_t}{\pi_{t+1}} \right]$$

$$+\lambda_{2t} \left[\frac{1}{p_t^*} - \left((1-\xi_p) \left(\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1 - \xi_p} \right)^{\lambda_f} + \frac{\xi_p \pi_t^{\frac{\lambda_f}{\lambda_f - 1}}}{p_{t-1}^*} \right) \right]$$

$$+\lambda_{3t} \left[1 + \beta \xi_p E_t \left(\frac{1}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}} F_{p,t+1} - F_{p,t} \right]$$

$$+\lambda_{4t} p_t^* \left[h_t z_h(h_t, \varsigma_t) + \beta \xi_p E_t \left(\frac{1}{\pi_{t+1}} \right)^{\frac{\lambda_f}{1-\lambda_f}} \frac{p_{t+1}^*}{p_t^*} \frac{K_{p,t+1}}{p_{t+1}^*} - \frac{K_{p,t}}{p_t^*} \right]$$

$$+\lambda_{5t} \left[F_{p,t} \left(\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1 - \xi_p} \right)^{1-\lambda_f} - K_{p,t} \right] \}.$$

In the fourth Lagrangian constraint, it is convenient to factor out p_t^*. We conjecture (and can later verify) that the first, third, fourth and fifth constraints are not binding on the problem. In particular, we can simply select R_t to satisfy the first constraint, $F_{p,t}$ to satisfy the third, $K_{p,t}$ to satisfy the fourth, and we then need to verify that the fifth constraint is satisfied, as well as $R_t \geq 1$.

Implementing the conjecture, the problem, with $g_t \equiv 0$, reduces to:

$$\max_{p_t^*, h_t, \pi_t} E_0 \sum_{t=0}^{\infty} \beta^t \{ \log p_t^* + \log h_t - z(h_t, \varsigma_t)$$

$$+\lambda_{2t} \left[\frac{1}{p_t^*} - \left((1-\xi_p) \left(\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1 - \xi_p} \right)^{\lambda_f} + \frac{\xi_p \pi_t^{\frac{\lambda_f}{\lambda_f - 1}}}{p_{t-1}^*} \right) \right] \},$$

where $\log A_t$ in the utility function is ignored because it cannot be controled. This leads to the efficiency condition for hours and p_t^*:

$$h_t z_h(h_t, \varsigma_t) = 1. \tag{A.18}$$

Interestingly, this coincides with the first-best setting for h_t. The efficiency conditions for p_t^*

and π_t are, respectively,

$$\begin{aligned} p_t^* &= \lambda_{2t} - \beta\lambda_{2t+1}\xi_p\pi_{t+1}^{\frac{\lambda_f}{\lambda_f-1}} = 0 \\ \left(\frac{1-\xi_p\pi_t^{\frac{1}{\lambda_f-1}}}{1-\xi_p}\right)^{\lambda_f-1} &= \frac{\pi_t}{p_{t-1}^*}. \end{aligned} \tag{A.19}$$

Rearranging, and substituting into (A.12), we obtain:

$$p_t^* = \left[\left(p_{t-1}^*\right)^{\frac{1}{\lambda_f-1}}\xi_p + \left(1-\xi_p\right)\right]^{\lambda_f-1} \tag{A.20}$$

$$\pi_t = \frac{p_{t-1}^*}{p_t^*}. \tag{A.21}$$

We now verify that all the constraints on the Ramsey problem assumed to be non-binding are in fact satisfied. Consider the fourth Lagrangian constraint, after substituting out (A.18), (A.20), and (A.21):

$$1 + \beta\xi_p E_t\left(\frac{1}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_f}}\frac{K_{p,t+1}}{p_{t+1}^*} = \frac{K_{p,t}}{p_t^*}.$$

We let this equation define $K_{p,t}$, so that the fourth Lagrangian is satisfied. The requirement that the third Lagrangian constraint also be satisfied implies

$$\frac{K_{p,t}}{p_t^*} = F_{p,t}. \tag{A.22}$$

Letting (A.22) define $F_{p,t}$ we have that the third Lagrangian is satisfied. From (A.19) and (A.21),

$$\left(\frac{1-\xi_p\pi_t^{\frac{1}{\lambda_f-1}}}{1-\xi_p}\right)^{\lambda_f-1} = p_t^*. \tag{A.23}$$

It follows from (A.22) and (A.23) that the fifth Lagrangian constraint is satisfied. The first Lagrangian constraint is trivially satisfy if we use it to define the nominal rate of interest, R_t. As long as the shocks are not too big, $R_t \geq 1$.

We have established that allocations in which hours worked, the price distortions and inflation satisfy (A.18), (A.20), and (A.21), respectively, solve the Ramsey problem. Because h_t is independent of A_t according to (A.18), it follows that the welfare cost of business cycles in the Ramsey problem is the same, regardless of whether the underlying economy is the standard DSGE model or our model of involuntary unemployment.

A.3.2. Taylor Rule Equilibrium

Consider the following policy rule:

$$R_t = R^{1-\rho_R}R_{t-1}^{\rho_R}\pi_t^{(1-\rho_R)r_\pi}x_t^{(1-\rho_R)r_y}, \tag{A.24}$$

where

$$x_t = \frac{Y_t}{\tilde{Y}_t},$$

and $\tilde{Y}_t$ is the first-best level of consumption. We can write

$$x_t = \frac{p_t^* A_t h_t}{A_t \tilde{h}_t} = \frac{p_t^* h_t}{\tilde{h}_t},$$

where h_t is equilibrium hours worked and $\tilde{h}_t$ is the value of hours worked in the efficient equilibrium, which is define by:

$$\frac{A_t}{\tilde{C}_t} = z_h\left(\tilde{h}_t, \varsigma_t\right).$$

Here, $\tilde{C}_t$ denotes consumption in the efficient equilibrium. Using the resource constraint with no price distortions, (A.7) with $p_t^* = 1$, and rewriting:

$$\frac{A_t}{A_t \tilde{h}_t - G_t} = \frac{1}{\tilde{h}_t - g_t n_t} = z_h\left(\tilde{h}_t, \varsigma_t\right),$$

or,

$$\left(\tilde{h}_t - g_t n_t\right) z_h\left(\tilde{h}_t, \varsigma_t\right) = 1.$$

Equilibrium is characterized by the requirement that (A.24) as well as the above equation be satisfied, and also (A.16), (A.17), (A.4), (A.9), (A.12), and (A.15). Summarizing:

$$0 = E_t\left[p_t^* h_t z_h\left(h_t, \varsigma_t\right) + \beta\xi_p\left(\frac{1}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} K_{p,t+1} - K_{p,t}\right] \tag{A.25}$$

$$0 = E_t\left[\frac{p_t^* h_t}{p_t^* h_t - g_t n_t} + \beta\xi_p E_t\left(\frac{1}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_f}} F_{p,t+1} - F_{p,t}\right] \tag{A.26}$$

$$K_{p,t} = \left[\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1 - \xi_p}\right]^{1-\lambda_f} F_{p,t} \tag{A.27}$$

$$p_t^* = \left[\left(1 - \xi_p\right)\left(\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1 - \xi_p}\right)^{\lambda_f} + \frac{\xi_p \pi_t^{\frac{\lambda_f}{\lambda_f - 1}}}{p_{t-1}^*}\right]^{-1} \tag{A.28}$$

$$1 = \beta E_t \frac{p_t^* h_t - g_t n_t}{\left(p_{t+1}^* h_{t+1} - g_{t+1} n_{t+1}\right) g_{A,t+1}} \frac{R_t}{\pi_{t+1}} \tag{A.29}$$

$$R_t = R^{1-\rho_R} R_{t-1}^{\rho_R} \pi_t^{(1-\rho_R) r_\pi}\left(\frac{p_t^* h_t}{\tilde{h}_t}\right)^{(1-\rho_R) r_y} \tag{A.30}$$

$$1 = \left(\tilde{h}_t - g_t n_t\right) z_h\left(\tilde{h}_t, \varsigma_t\right) \tag{A.31}$$

$$n_t = \left(\frac{n_{t-1}}{g_{A,t}}\right)^{1-\gamma} \tag{A.32}$$

The 8 variables to be determined by these 8 equations are:

$$K_{p,t}, F_{p,t}, h_t, \tilde{h}_t, p_t^*, \pi_t, R_t, n_t \tag{A.33}$$

The present discounted value of utility is:

$$\begin{aligned} & E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - z\left(h_t, \varsigma_t\right)\right] \qquad \text{(A.34)} \\ = \; & E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(p_t^* h_t - g_t n_t\right) - z\left(h_t, \varsigma_t\right)\right] + \frac{\log A_0}{1-\beta} + \frac{1}{1-\beta} E_0 \sum_{t=1}^{\infty} \beta^t \log g_{A,t} \end{aligned}$$

The piece that is exogenous and additive is not interesting. Define

$$\begin{aligned} w\left(g_{A,0}, p_{-1}^*, \varsigma_0\right) &= E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(p_t^* h_t - g_t n_t\right) - z\left(h_t, \varsigma_t\right)\right] \\ &= \log\left(p_0^* h_0 - g_0 n_0\right) - z\left(h_0, \varsigma_0\right) + \beta E_0 w\left(g_{A,1}, p_0^*, \varsigma_1\right). \end{aligned}$$

We can add this equation to the list, (A.25)-(A.30), above,

$$E_t\left[\log\left(p_t^* h_t - g_t n_t\right) - z\left(h_t, \varsigma_t\right) + \beta w_{t+1} - w_t\right] = 0, \tag{A.35}$$

giving us one additional variable, w_t, and one additional equation.

This system can be solved in Dynare. Simply type in equations (A.25)-(A.32) and (A.35). We have three stochastic processes, g_t, $g_{A,t}$ and ς_t. Let x_t denote one of these. Then, the law of motion of x_t is:

$$\log x_t = (1-\rho_x)\log x + \rho_x \log x_{t-1} + \varepsilon_t^x,$$

for $x_t = g_t$, $g_{A,t}$, ς_t.

A.4. Steady State

From (A.5) and (A.6),

$$K_p = F_p,\; s = 1/\lambda_f,\; F_p = \frac{\frac{1}{1-\eta_g}}{1-\beta\xi_p}.$$

From (A.12), in a zero inflation steady state,

$$p^* = 1.$$

We want to impose that government spending is a given proportion, η_g, of total output in the steady state:

$$\eta_g = \frac{G}{C+G} = \frac{gnA}{Ah} = \frac{gn}{h} = \frac{g\left(\frac{1}{g_A}\right)^{\frac{1-\gamma}{\gamma}}}{h},$$

by (A.10). Thus, the steady state value of g is:

$$g = \eta_g h g_A^{\frac{1-\gamma}{\gamma}}, \; gn = \eta_g h \tag{A.36}$$

Combining (A.13), the result for the steady state value of s and (A.7), (A.11) to obtain:

$$\frac{h}{h - gn} = h z_h (h, \varsigma),$$

or, using (A.36),

$$1 = \left[1 - \eta_g\right] h z_h (h, \varsigma). \tag{A.37}$$

We now proceed to develop the formulas necessary to compute z_h. Recall, the disutility of labor function, $z(h_t)$ in (2.22). Write this in terms of m_t :

$$Z(m_t) = \log\left[Q(m_t)\left(e^{F+\varsigma_t(1+\sigma_L)m_t^{\sigma_L}} - 1\right) + 1\right] - \frac{a^2\varsigma_t^2(1+\sigma_L)\sigma_L^2}{2\sigma_L+1} m_t^{2\sigma_L+1} - \eta\varsigma_t\sigma_L m_t^{\sigma_L+1}, \tag{A.38}$$

where

$$\begin{aligned} h_t &= m_t\eta + a^2\varsigma_t\sigma_L m_t^{\sigma_L+1} \equiv Q(m_t), \\ m_t &= Q^{-1}(h_t), \end{aligned} \tag{A.39}$$

where Q^{-1} is inverse function of Q, defined by:

$$h_t = Q\left(Q^{-1}(h_t)\right). \tag{A.40}$$

Then,

$$z(h_t) = Z\left(Q^{-1}(h_t)\right). \tag{A.41}$$

We require the first and second derivatives of z. Thus,

$$z_h(h_t) = Z_m\left(Q^{-1}(h_t)\right)\left[Q^{-1}\right]'(h_t), \tag{A.42}$$

where $[Q^{-1}]'$ denotes the derivative of the function, Q^{-1}. To obtain an expression for $[Q^{-1}]'(h_t)$, differentiate (A.40) with respect to h_t :

$$1 = Q'\left(Q^{-1}(h_t)\right)\left[Q^{-1}\right]'(h_t),$$

so that:

$$\left[Q^{-1}\right]'(h_t) = \frac{1}{Q'\left(Q^{-1}(h_t)\right)}. \tag{A.43}$$

We also require the second derivative of Q^{-1}. Differentiating (A.40) a second time with respect to h_t :

$$0 = Q''\left(Q^{-1}(h_t)\right)\left(\left[Q^{-1}\right]'(h_t)\right)^2 + Q'\left(Q^{-1}(h_t)\right)\left[Q^{-1}\right]''(h_t).$$

Substituting from the first derivative:

$$0 = Q''\left(Q^{-1}\left(h_t\right)\right)\left(\frac{1}{Q'\left(Q^{-1}\left(h_t\right)\right)}\right)^2 + Q'\left(Q^{-1}\left(h_t\right)\right)\left[Q^{-1}\right]''\left(h_t\right),$$

so that

$$\left[Q^{-1}\right]''\left(h_t\right) = -Q''\left(Q^{-1}\left(h_t\right)\right)\left(\frac{1}{Q'\left(Q^{-1}\left(h_t\right)\right)}\right)^3. \tag{A.44}$$

Substituting (A.43) into (A.42),

$$z_h\left(h_t\right) = \frac{Z_m\left(Q^{-1}\left(h_t\right)\right)}{Q'\left(Q^{-1}\left(h_t\right)\right)}.$$

Differentiating z in (A.41) a second time,

$$z_{hh}\left(h_t\right) = Z_{mm}\left(Q^{-1}\left(h_t\right)\right)\left(\left[Q^{-1}\right]'\left(h_t\right)\right)^2 + Z_m\left(Q^{-1}\left(h_t\right)\right)\left[Q^{-1}\right]''\left(h_t\right),$$

which, after substituting from (A.43) and (A.44), is:

$$\begin{aligned} z_{hh}\left(h_t\right) &= Z_{mm}\left(Q^{-1}\left(h_t\right)\right)\left(\frac{1}{Q'\left(Q^{-1}\left(h_t\right)\right)}\right)^2 - Z_m\left(Q^{-1}\left(h_t\right)\right)Q''\left(Q^{-1}\left(h_t\right)\right)\left(\frac{1}{Q'\left(Q^{-1}\left(h_t\right)\right)}\right)^3 \\ &= \left(\frac{1}{Q'\left(Q^{-1}\left(h_t\right)\right)}\right)^2 Z_m\left(Q^{-1}\left(h_t\right)\right)\left[\frac{Z_{mm}\left(Q^{-1}\left(h_t\right)\right)}{Z_m\left(Q^{-1}\left(h_t\right)\right)} - \frac{Q''\left(Q^{-1}\left(h_t\right)\right)}{Q'\left(Q^{-1}\left(h_t\right)\right)}\right] \\ &= \left(\frac{1}{Q'\left(m_t\right)}\right)^2 Z_m\left(m_t\right)\left[\frac{Z_{mm}\left(m_t\right)}{Z_m\left(m_t\right)} - \frac{Q''\left(m_t\right)}{Q'\left(m_t\right)}\right], \end{aligned}$$

where the expressions for Q', Q'', Z_m, Z_{mm} can be obtained by symbolic differentiation of the underlying functions, (A.38) and (A.39).

The endogenous variables are

$$h, m, \sigma_z, u, \bar{p},$$

and the equations are:

$$\begin{aligned} (1)h &= m\eta + a^2\varsigma\sigma_L m^{\sigma_L+1} \\ (2)\bar{p} &= \eta + \varsigma a^2\left(1+\sigma_L\right)m^{\sigma_L} \\ (3)1 &= \left[1-\eta_g\right]hz_h\left(h,\varsigma\right) \\ (4)\sigma_z &= \frac{z_{hh}h}{z_h} \\ (5)\kappa^{okun} &= \frac{a^2\varsigma\sigma_L^2 m^{\sigma_L}\left(1-u\right)}{1-u+a^2\varsigma\sigma_L^2 m^{\sigma_L}} \\ u &= \frac{m-h}{m}. \end{aligned} \tag{A.45}$$

The parameters are:

$$F, \varsigma, a, \eta, \sigma_L.$$

We solve the steady state using two different strategies, depending on our purposes. The first strategy, the straightforward one, takes the parameters as given and computes the steady values of the endogenous variables. We implement this strategy by placing a grid of values of m on the unit interval. For each value of m on the grid we compute h using (1) and evaluate (3). We count the number of places on the grid where there is a switch in the sign of (3), *i.e.*, in $1 - \left[1 - \eta_g\right] hz_h\left(h, \varsigma\right).$ For each sign switch, we then narrowed down the corresponding value of m that implements (3) exactly using a nonlinear equation solver. After this, we computed $\bar{p}$ using (2) as well as steady state unemployment, u. In practice, we found two sets of solutions to the steady state equations, but one was inadmissible because it implied $u < 0$. We applied this 'first strategy' to compare the steady states of the involuntary unemployment model studied in this section with the steady state of the full information model in section B holding model parameters fixed. Section B shows that the steady state of the full information model is also characterized by (A.45). The difference between the two models lies in the details of the function, z_h. With this approach to computing the steady state we are able to evaluate the impact on welfare and other variables of the assumption of limited information. We compute the value of information in consumption units as follows. According to (A.34), steady state utility is, apart from an additive constant, as follows:

$$U = \frac{1}{1-\beta}\left[\log\left(h\right) - z\right].$$

Here, h denotes steady state employment and z denotes the steady state disutility of labor (the fact, $h - gn = \left(1 - \eta_g\right) h$, in steady state has been used here). Let utility in the steady state of the full information model be denoted:

$$U^{fi} = \frac{1}{1-\beta}\left[\log\left(h^{fi}\right) - z^{fi}\right]$$

Let λ denote the percent increase in consumption in the involuntary unemployment model that makes households in that model indifferent between staying in that environment, or converting to the environment with full information. Let $U\left(\lambda\right)$ denote the level of utility in the involuntary unemployment equilibrium when consumption is raised from h to $h\left(1 + \lambda/100\right).$ We seek λ such that

$$U\left(\lambda\right) = U^{fi}.$$

Note:

$$\begin{aligned} U\left(\lambda\right) &= \frac{1}{1-\beta}\left[\log\left(h\right) + \log\left(1 + \lambda/100\right) - z\right] \\ &= U + \frac{1}{1-\beta}\log\left(1 + \lambda/100\right), \end{aligned}$$

so that

$$\lambda = 100\left[e^{(1-\beta)\left(U^{fi} - U\right)} - 1\right].$$

Our second strategy for computing the steady state takes the values of $\bar{p}$, m, u and σ_z as given. Thus, the 'parameters' become:

$$\bar{p}, m, u, \sigma_z, \sigma_L \tag{A.46}$$

and the 'endogenous' variables become:

$$F, \varsigma, a, \eta, h.$$

We wish to solve the system for the endogenous variables, for a given set of values for the parameters. To do so, note first that h is determined by

$$h = m(1-u).$$

We then proceed to solve equations (1)-(4) with respect to F, ς, a, η using a nonlinear multipler equation solver. This solution strategy is useful when we want to compute observationally equivalent parameterizations for the models considered in this paper. When we do this, we take observed data as given. Variables like m and u are measured from time averages of unemployment and the labor force, and σ_z can be estimated from applying time series techniques to the reduced form of the model using macroeconomic data that do not include unemployment and the labor force.

In our analysis, we find it convenient to compare the model with involuntary unemployment with two other models. This includes the CGG model and the model in which the family problem is based on full information. The lattter is treated in the section devoted to that model. Here, we develop the steady state equations for the CGG model.

The only change required for computing the steady state lies in the specification of z_h in (A.37). We define the disutility of labor in the CGG model as the one implicit in the 'standard model', (3.21):

$$z(h_t, \varsigma_t) = \varsigma h_t^{1+\sigma_L}.$$

Then, (A.37) reduces to:

$$1 = \left[1-\eta_g\right] h z_h(h, \varsigma) = \left[1-\eta_g\right] \varsigma (1+\sigma_L) h^{\sigma_L+1}.$$

Note that as η_g increases, h does too. In the CGG model $m = h$ because there is no job search.

We follow the approach taken in the involuntary unemployment model in calibrating m, and thus h too (see (A.46)). We thus define the preference shock in this model as

$$\varsigma = \frac{1}{\left[1-\eta_g\right] (1+\sigma_L) h^{1+\sigma_L}}.$$

Of course the steady state unemployment rate is zero (as it is outside of steady state too) because there is no monopoly power in the labor market. In addition, we parameterizer the curvature, σ_z, of the disutility of labor. This is simply

$$\sigma_L = \sigma_z$$

in the CGG model. Then, labor supply is simply:

$$z_h(h_t, \varsigma_t) = \varsigma(1 + \sigma_L) h_t^{\sigma_L},$$

B. The Family Utility Function Under Full Information

Our model of involuntary unemployment makes two sorts of assumptions: (i) households have different aversions to work and must make an effort to find work and (ii) their type and effort levels are private information. The fact that there is unemployment in the BLS sense follows from (i) only and (ii) is required for unemployment to be 'involuntary'. This section allows us to determine the impact on the analysis of (ii), by deriving the family utility function that applies when (ii) is not satisfied.

The first subsection derives the family utility function. The second subsection discusses the steady state when this model of unemployment is introduced into CGG.

B.1. Family Utility Function

Thus, we assume that the family observes everything about the individual household: the effort it exerts to find a job, if any, and its aversion to work. The family selects a consumption allocation and level of search effort, conditional on a household's type. It does so by optimizing the ex ante utility of an arbitrary household or, equivalently, by optimizing the average utility of all households ex post. Households with $0 \leq l \leq m$ participate in the labor force and those with $1 \geq l \geq m$ do not, where m is a choice variable. We drop subscripts to simplify the notation. The family optimization problem is:

$$\max_{m,\{e_l\},c^w,c^{nw}} \int_0^m \left(p(e_l)\left[\log(c^w) - F - \varsigma(1+\sigma_L) l^{\sigma_L}\right] + (1 - p(e_l))\log(c^{nw}) - \frac{1}{2}e_l^2 \right) dl$$
$$+ (1 - m)\log(c^{nw})$$

subject to the resource constraint, (2.18), and the link between the number employed, h, and the labor force, m, (2.11). We reproduce these constraints here for convenience:

$$\begin{aligned} C &= hc^w + (1-h)c^{nw} \\ h &= \int_0^m p(e_l)\, dl = m\eta + a \int_0^m e_l dl. \end{aligned}$$

Rewrite the objective:

$$\max_{m,\{e_l\},c^w,c^{nw}} \int_0^m \left(p(e_l) \left[\log\left(\frac{c^w}{c^{nw}}\right) - F - \varsigma (1+\sigma_L) l^{\sigma_L} \right] - \frac{1}{2} e_l^2 \right) dl + \log(c^{nw}).$$

From the resource constraint:

$$\frac{c^w}{c^{nw}} = \frac{\frac{C}{c^{nw}} - (1-h)}{h}.$$

So that, after substituting out the resource constraint, we get:

$$\max_{m,\{e_l\},c^{nw}} \int_0^m \left(p(e_l) \left[\log\left(\frac{\frac{C}{c^{nw}} - (1-h)}{h}\right) - F - \varsigma (1+\sigma_L) l^{\sigma_L} \right] - \frac{1}{2} e_l^2 \right) dl + \log(c^{nw})$$
$$+\lambda \left[\int_0^m p(e_l)\, dl - h \right],$$

where λ is the multiplier on the restriction linking m and h. The first order condition for c^{nw} is:

$$c^{nw} = C,$$

so that the objective reduces to:

$$\max_{m,\{e_l\}} \int_0^m \left(-p(e_l) \left[F + \varsigma (1+\sigma_L) l^{\sigma_L} \right] - \frac{1}{2} e_l^2 \right) dl + \log(C) \tag{B.1}$$
$$+\lambda \left[\int_0^m p(e_l)\, dl - h \right].$$

Optimization with respect to e_l implies:

$$e_l = \lambda a - a \left[F + \varsigma (1+\sigma_L) l^{\sigma_L} \right],$$

so that, using (2.5),

$$p(e_l) = \eta + \lambda a^2 - a^2 \left[F + \varsigma (1+\sigma_L) l^{\sigma_L} \right].$$

Also,

$$h = \int_0^m p(e_l)\, dl = m \left(\eta + \lambda a^2 - a^2 \left[F + \varsigma m^{\sigma_L} \right] \right), \tag{B.2}$$

or,

$$\lambda = \frac{\frac{h}{m} - \eta}{a^2} + F + \varsigma m^{\sigma_L}. \tag{B.3}$$

Optimality of the choice of m in (B.1) implies, by Leibniz's rule:

$$-p(e_m) \left[F + \varsigma (1+\sigma_L) m^{\sigma_L} \right] - \frac{1}{2} e_m^2 + \lambda p(e_m) = 0,$$

or,

$$-\left[\eta + \lambda a^2 - a^2 x \right] x - \frac{1}{2} (\lambda a - a x)^2 + \lambda \left(\eta + \lambda a^2 - a^2 x \right) = 0,$$

where

$$x \equiv F + \varsigma \left(1 + \sigma_L\right) m^{\sigma_L}. \tag{B.4}$$

Simplifying,

$$-\left(\eta + \lambda a^2\right) x + a^2 x^2 - \frac{1}{2} \left(\lambda a\right)^2 + \lambda a^2 x - \frac{1}{2} a^2 x^2 + \lambda \left(\eta + \lambda a^2\right) - \lambda a^2 x = 0,$$

or,

$$f\left(x\right) \equiv x^2 - 2\left(\frac{\eta + \lambda a^2}{a^2}\right) x + \lambda \left[\lambda + 2\frac{\eta}{a^2}\right] = 0. \tag{B.5}$$

It is easy to verify that there are two values of x with the property, $f\left(x\right) = 0$:

$$\frac{1}{a^2}\left(\lambda a^2 + 2\eta\right),\ \lambda.$$

Substituting out for λ from (B.3) these solutions reduce to:

$$\frac{\frac{h}{m} + \eta}{a^2} + F + \varsigma m^{\sigma_L},\ \frac{\frac{h}{m} - \eta}{a^2} + F + \varsigma m^{\sigma_L}.$$

Consider the first solution:

$$F + \varsigma \left(1 + \sigma_L\right) m^{\sigma_L} = \frac{\frac{h}{m} + \eta}{a^2} + F + \varsigma m^{\sigma_L}$$

or,

$$m^{\sigma_L} = \frac{\frac{h}{m} + \eta}{a^2 \varsigma \sigma_L} \tag{B.6}$$

There is a unique value of m, $m \geq 0$, that satisfies (B.6). To see this, note that the left side of (B.6) starts at zero and increases without bound as m increases. The right side starts at plus infinity (thus, greater than the left size) with $m = 0$ and declines monotonically to a finite number as m increases (thus, the right side is eventually below the left side). By continuity and monotonicity, there is a unique value of m that satisfies the equality in (B.6). Of course, the only admissible solution satisfies $m \in \left(h, 1\right)$. The second solution to $f\left(x\right) = 0$ implies

$$F + \varsigma \left(1 + \sigma_L\right) m^{\sigma_L} = \frac{\frac{h}{m} - \eta}{a^2} + F + \varsigma m^{\sigma_L},$$

or

$$m^{\sigma_L} = \frac{\frac{h}{m} - \eta}{a^2 \varsigma \sigma_L}. \tag{B.7}$$

Interestingly, this also reduces to the expression in (A.39):

$$h = \eta m + a^2 \varsigma \sigma_L m^{\sigma_L + 1} \equiv Q\left(m\right). \tag{B.8}$$

There is a unique value of m, $m \geq 0$, that satisfies (B.7) for any $h \geq 0$. This is because m is monotone increasing for $m \geq 0$ and $Q\left(0\right) = 0$. Whether one or both of (B.6) and (B.7) correspond to local maxima requires examining the relevant second order condition. We do so now, graphically. Of course, we can anticipate that the smaller of the two solutions, the one associated with (B.7), is likely to correspond to the maximum sought in (B.1).

The existence of more than one solution to (B.5) implies that we must investigate second order conditions. Differentiate (B.5) with respect to m :

$$f'(x)\frac{dx}{dm} = \left[x - \frac{\eta + \lambda a^2}{a^2}\right] 2\frac{dx}{dm}.$$

Since $2dx/dm > 0$ the sign of the above expression corresponds to the sign of the object in square brackets, which is, after substituting out for x from (B.4) and for λ from (B.3):

$$\varsigma \sigma_L m^{\sigma_L} - \frac{\frac{h}{m}}{a^2}$$

It is easy to verify that (B.6) implies the above expression is positive, while (B.7) implies the above expression is negative. Thus, (B.7) satisfies the first and second order conditions necessary for an optimum. Thus, (B.7) is a local optimum, while (B.6) is a local minimum.

It is interesting to investigate whether (B.9) satisfies the usual bounds for a probability, given (B.3) and (B.7). Using (B.3) to substitute out for λ in (B.9) and rearranging,

$$p(e_l) = \frac{h}{m} + a^2 \varsigma \left[m^{\sigma_L} - (1 + \sigma_L) l^{\sigma_L}\right].$$

Using (B.7) to substitute out for h/m in the previous expression, we obtain:

$$p(e_l) = \eta + (1 + \sigma_L) a^2 \varsigma \left[m^{\sigma_L} - l^{\sigma_L}\right]. \tag{B.9}$$

Interestingly, this is the same function obtained for the involuntary unemployment model (to see this, substitute the incentive constraint, (2.9), into the probability function for that model, (2.6)).

We would like to have an expression for the family utility function, the function that attains the optimum in (B.1) for given C and h. Consider the object under the integral in the objective function, (B.1):

$$
\begin{aligned}
& -p\left(e_l\right)\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]-\frac{1}{2} e_l^2 \\
= & -\left[\eta+\lambda a^2-a^2\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]\right]\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]-\frac{1}{2}\left[\lambda a-a\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]\right]^2 \\
= & -\left(\eta+\lambda a^2\right)\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]+a^2\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]^2-\frac{1}{2}\left[\lambda a-a\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]\right]^2 \\
= & -\left(\eta+\lambda a^2\right)\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]+a^2\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]^2 \\
& -\frac{1}{2}\left[(\lambda a)^2-2 \lambda a^2\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]+a^2\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]^2\right] \\
= & -\left(\eta+\lambda a^2\right)\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]+a^2\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]^2 \\
& -\frac{1}{2}(\lambda a)^2+\lambda a^2\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]-\frac{1}{2} a^2\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]^2 \\
= & -\eta\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]+\frac{1}{2} a^2\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]^2-\frac{1}{2}(\lambda a)^2
\end{aligned}
$$

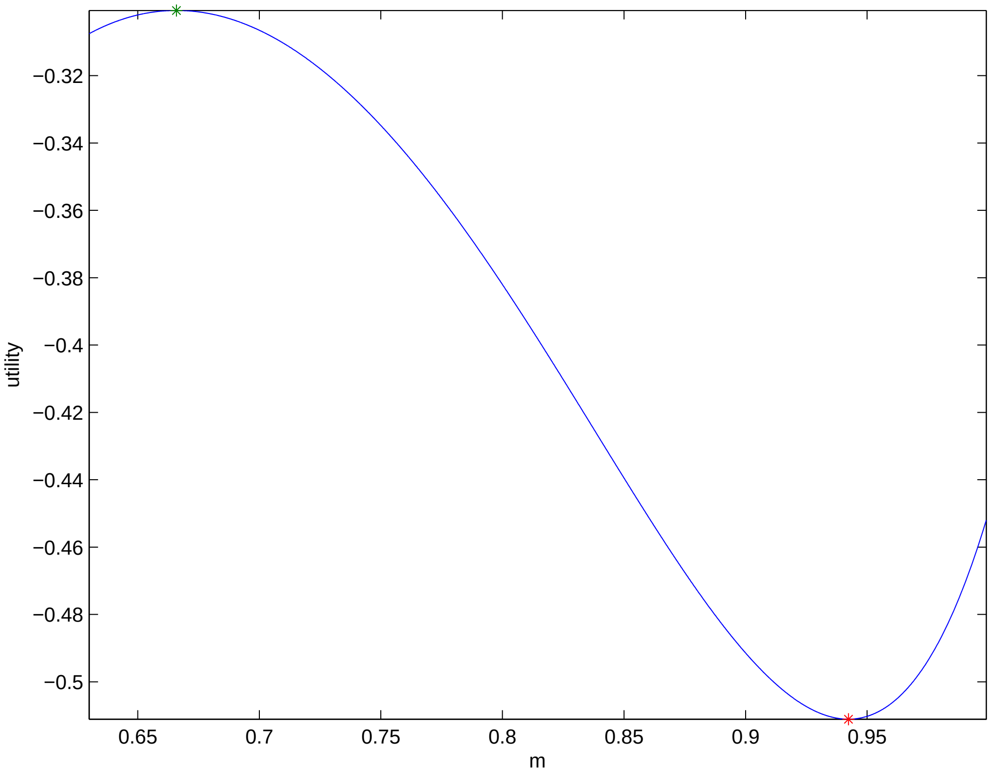
Note,

$$
\begin{aligned}
\int_0^m\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right] d l & =F m+\varsigma m^{\sigma_L+1} \\
\int_0^m\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]^2 d l & =\int_0^m\left[F^2+2 \varsigma\left(1+\sigma_L\right) l^{\sigma_L} F+\varsigma^2\left(1+\sigma_L\right)^2 l^{2 \sigma_L}\right] d l \\
& =m F^2+2 \varsigma m^{\sigma_L+1} F+\varsigma^2\left(1+\sigma_L\right)^2 \frac{m^{2 \sigma_L+1}}{2 \sigma_L+1}
\end{aligned}
$$

Then, the integral in (B.1) is

$$
\begin{aligned}
& \int_0^m\left\{-\eta\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]+\frac{1}{2} a^2\left[F+\varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right]^2-\frac{1}{2}(\lambda a)^2\right\} d l \\
= & -\eta m\left[F+\varsigma m^{\sigma_L}\right]+\frac{1}{2} a^2 m\left[F^2+2 \varsigma m^{\sigma_L} F+\varsigma^2\left(1+\sigma_L\right)^2 \frac{m^{2 \sigma_L}}{2 \sigma_L+1}\right]-\frac{1}{2}(\lambda a)^2 m,
\end{aligned}
$$

with λ given by (B.3) and m given by (B.6). The graph of this object, for $m \in (h, 1)$ is as follows:



Note that there are two values of m where this function is flat. They correspond to (B.6) and (B.7). Consistent with the algebra above, the second flat point is a local minimum, while the first is a local maximum.

To construct the utility function of the representative agent, write the following function of m :

$$Z(m,h) = \eta m \left[F + \varsigma m^{\sigma_L}\right] - \frac{1}{2}a^2 m \left[F^2 + 2\varsigma m^{\sigma_L} F + \varsigma^2 (1+\sigma_L)^2 \frac{m^{2\sigma_L}}{2\sigma_L + 1}\right] + \frac{1}{2}(\lambda(m,h)a)^2 m, \tag{B.10}$$

where $\lambda(m,h)$ is the particular function of m and h given by (B.3). Express (B.8) as

$$m = Q^{-1}(h). \tag{B.11}$$

Then, the utility function of the representative agent is:

$$u(C,h) = \log(C) - z(h), \tag{B.12}$$

where

$$z(h) = Z\left(Q^{-1}(h), h\right), \tag{B.13}$$

where Q^{-1} is defined as the inverser function of Q (see (A.40)).

Our calculations require the derivative of z, $z_h(h)$. According to (B.13),

$$z_h(h) = Z_1\left(Q^{-1}(h), h\right)\left[Q^{-1}\right]'(h) + Z_2\left(Q^{-1}(h), h\right),$$

where $\left[Q^{-1}\right]'(h)$ is the derivative of the inverse function of Q, defined in terms of Q' in (A.43). Using (A.43):

$$z_h(h) = \frac{Z_1\left(Q^{-1}(h), h\right)}{Q'\left(Q^{-1}(h_t)\right)} + Z_2\left(Q^{-1}(h), h\right). \tag{B.14}$$

We also require the second derivative of z, z_{hh}:

$$\begin{aligned} z_{hh}(h) &= Z_{11}\left(Q^{-1}(h), h\right)\left(\left[Q^{-1}\right]'(h)\right)^2 + Z_{12}\left(Q^{-1}(h), h\right)\left[Q^{-1}\right]'(h) \\ &\quad + Z_1\left(Q^{-1}(h), h\right)\left[Q^{-1}\right]''(h) + Z_{21}\left(Q^{-1}(h), h\right)\left[Q^{-1}\right]'(h) \\ &\quad + Z_{22}\left(Q^{-1}(h), h\right), \end{aligned}$$

where the second derivative of the inverse function, $\left[Q^{-1}\right]''(h)$, can be computed using (A.44). Using (A.43) and (A.44):

$$\begin{aligned} z_{hh}(h) &= Z_{11}\left(Q^{-1}(h), h\right)\left(\frac{1}{Q'\left(Q^{-1}(h_t)\right)}\right)^2 + \frac{Z_{12}\left(Q^{-1}(h), h\right)}{Q'\left(Q^{-1}(h_t)\right)} \\ &\quad - Z_1\left(Q^{-1}(h), h\right)Q''\left(Q^{-1}(h_t)\right)\left(\frac{1}{Q'\left(Q^{-1}(h_t)\right)}\right)^3 \\ &\quad + \frac{Z_{21}\left(Q^{-1}(h), h\right)}{Q'\left(Q^{-1}(h_t)\right)} + Z_{22}\left(Q^{-1}(h), h\right), \end{aligned} \tag{B.15}$$

Given (B.14) and (B.14), we can compute the curvature of utility:

$$\sigma_z = \frac{z_{hh}h}{z_h}.$$

All these formulas can be computed symbolically given the definitions of the Z and Q functions in (B.10) and (B.8), respectively.

B.2. Steady State

We discuss two ways of computing the steady state of the model. The first is the 'natural' one. This takes the parameters of the model as given and computes the implied exogenous variables. This is useful for doing one type of comparison of the full information version of the model with our involuntary unemployment model. For example, we can ask how the level of unemployment is affected by the absence of full information. The second approach allows us to impose certain endogenous variables exogenously, and back out parameters. This approach is useful when we want to ask what an econometrician observing data (and, hence given endogenous variables) would infer about the welfare cost of business cycles.

B.2.1. A First Approach to Computing the Steady State

The structural parameters of the model are

$$F, \varsigma, a, \eta, \sigma_L.$$

The endogenous variables of interest are

$$\bar{p}, h, m, u, \sigma_z.$$

The relevant equations are (A.45):

$$\begin{aligned}
(1)h &= m\eta + a^2 \varsigma \sigma_L m^{\sigma_L + 1} \\
(2)\bar{p} &= \eta + \varsigma a^2 (1 + \sigma_L) m^{\sigma_L} \\
(3)1 &= \left[1 - \eta_g\right] h z_h (h, \varsigma) \\
(4)\sigma_z &= \frac{z_{hh} h}{z_h} \\
u &= \frac{m - h}{m}.
\end{aligned}$$

Notably, equations (1) and (2) in (A.45) are literally unchanged from what they are in the full information model because the expressions for $p(e_l)$ and the mapping from m to h are unchanged. In addition, equations (3) and (4) are formally the same, although the details of z_h and z_{hh} are different for the partial information model because z is different. The steady state is found by solving (3) for h. Then, (1) is solved for m and (2) is solved for $\bar{p}$. Finally, the last equation is solved for u.

B.2.2. A Second Approach to Solving the Steady State

We proceed here in a way that is parallel to the approach taken in the involuntary unemployment model, in section A.4. In particular, the exogenous 'parameters' are (A.46):

$$\bar{p}, m, u, \sigma_z, \sigma_L.$$

treated asand the 'endogenous' variables are:

$$F, \varsigma, a, \eta, h.$$

As in A.4, h is derived trivially from the parameters:

$$h = (1 - u) m.$$

There now remain four endogenous variables to be solved for, F, ς, a, η, using (1)-(4). As before, we select F, ς, a, η to set these equations to zero for given values of the exogenous parameters.

C. Quantitative Properties of the Model Without Capital

This section explores various quantitative properties of our simple model without capital. The assumption of limited information lies at the heart of our model of unemployment, and we quantity the impact of this assumption by comparing the properties of the model with and without limited information. We also address a question raised by Lucas', analysis of the welfare cost of business cycles. Lucas used aggregate data and the assumption that insurance markets are perfect to argue that the welfare cost of business cycles is at most very small. In reality insurance markets are of course not perfect, but Lucas conjectured that this failure of his assumption does not have a first-order impact on his conclusions. We explore the validity of Lucas' conjecture in our model. We generate data from our involuntary unemployment model and investigate the impact on the estimate of the welfare cost of business cycles of wrongly assuming the existence of perfect insurance. The experiments we perform to reach this conclusion are preliminary and we indicate what it would take to do a full analysis. For example, a more reliable calculation would be one done on the model that we fit to the US data. However, we ran into a technical difficulty with this model. The first order condition for wage setting in that model takes the form of an infinite-ordered present value. To work with it, the expression must be written in recursive form. We were not able to express the actual non-linear first order condition in recursive form, though we can do so after it is linearized. The problem is that the welfare calculations require second order approximations. This is why we do not report the results of welfare calculations in our medium-sized DSGE model. We do, however, report calculations based on the model without capital, and our provisional finding is that Lucas' assumption of perfect insurance does not substantially distort his conclusions.

C.1. Parameter Values

To do the numerical calculations, we must assign values to our involuntary unemployment model parameters. The laws of motion of the exogenous shocks are as follows. Let x_t denote the shock. Then,

$$\log x_t = (1 - \rho_x) \log x + \rho_x \log x_{t-1} + \varepsilon_t^x, \; E\left(\varepsilon_t^x\right)^2 = (\sigma_x)^2,$$

for $x = g, g_A, \varsigma$. The monetary policy shock, ε_t, is assumed to be an iid process with standard deviation denoted σ_R. Table A1 reports the values of a subset of parameters that are relatively uncontroversial. For example, we set values for the Taylor rule that imply the Taylor principle is satisfied (i.e., $r_\pi > 1$), we assume substantial interest rate smoothing (ρ_R is large) and the feedback on the output gap is modest (r_y is small). The steady state share, η_g, of government consumption to gross output is 20 percent. Government consumption is a long weighted average of current and past technology (i.e., γ is close to zero). The autocorrelation of the three shocks in the model is large. Finally, the innovation standard deviation of the three shocks was chosen so that each shock, when operative in isolation, causes the standard deviation of quarterly output growth in the model to be around 1 percent, the corresponding post WWII average in US data. The exception is the standard deviation of the preference shock which is set to zero throughout the analysis in this section.

We treated the structural parameters associated with households' job finding function, $p(e)$, and their disutility of labor differently. These parameters are:

$$F,\ \varsigma,\ a,\ \eta,\ \sigma_L. \tag{C.1}$$

We do not have direct observations on these parameters, nor are we aware of any estimates of these parameters in the literature. So, we chose values for them so that, conditional on the value of η_g given in Table A1, the steady state of the involuntary unemployment model implies the values of

$$m,\ u,\ 1/\sigma_z,\ 1/\kappa^{okun},\ \bar{p} \tag{C.2}$$

that are reported in the top left panel of Table A2. The indicated steady state value of the labor force participation rate, $m = 2/3$, corresponds to the average value of this variable in recent years. The steady state value of unemployment, $u = 0.056$, corresponds to the average value of unemployment in the post war US data. The compensated family labor supply elasticity, $1/\sigma_z = 2$, is roughly the corresponding object used in the real business cycle literature.[41] This is where the distinction between the compensated family labor supply elasticity and the Frisch elasticity is important. If $1/\sigma_z$ were interpretable as a Frisch elasticity, then the labor literature implies a value of $1/\sigma_z$ well below unity. However, as noted above, $1/\sigma_z$ has no connection to any individual agent's willingness to vary hours worked in response to a wage change in our model. The value of κ^{okun} was selected so that the model is consistent with a standard estimate of Okun's law. Finally, we chose a value for $\bar{p}$ to ensure that in the stochastic version of the model, the likelihood of violating the upper bound constraint on the job finding probability is small. The mapping from (C.2) to (C.1) using the steady state equilibrium conditions is described in detail in section A.4 of this technical appendix.

[41] A standard real business cycle model (see, e.g., the 'divisible labor' model in Christiano and Eichenbaum, 1992) uses preferences, $\sum_{t=0}^{\infty} \beta^t \left[\log(C_t) + \psi(1-h_t)\right]$, where h_t denotes time worked of the representative agent, as a fraction of available time. The labor first order condition associated with the agent's optimal labor choice is $\psi C_t/(1-h_t) = w_t$, where w_t denotes the real wage. The consumption compensated (actually 'consumption constant', with these preferences) labor supply function is (apart from a constant), $log(1-h_t) = \log w_t$. This implies the following steady state elasticity of employment with respect to the wage:

$$\frac{d\log h_t}{d\log w_t} = -\frac{1-h}{h} = -2.$$

Here, we assume that in steady state, 1/3 of available time is devoted to market work, i.e., $h = 1/3$.

C.2. Welfare

Following is a brief discussion of the welfare calculations that we report below. The approach is standard, and we discuss it here only for completeness. Let

$$W\left(g_{A,t}, p_{t-1}^{*}, \varsigma_t, g_t, \varepsilon_t, \sigma\right)$$

denote the welfare of the representative family in our model, conditional on the period t state, $g_{A,t}$, p_{t-1}^{*}, ς_t, g_t, ε_t. Here, $g_{A,t}$ denotes the period t realization of the growth rate of technology, the previous period's measure of price distortions is p_{t-1}^{*} and the current realization of the aggregate utility shock is ς_t. Finally, ε_t denotes the period t realization of the monetary policy shock. The parameter, σ, is a scalar that multiplies the 3×1 vector composed of ε_t and the innovations to $g_{A,t}$ and ς_t. Thus, $\sigma = 0$ corresponds to the nonstochastic version of the model and $\sigma = 1$ corresponds to the stochastic version. We think of σ as a continuous variable, $0 \leq \sigma \leq 1$. In the numerical examples considered, we allow only one shock to be stochastic at a time. We always set the preference shock, ς_t, to a constant.

We define the welfare cost of business cycles as

$$\Delta = W\left(g_{A,t}, p_{t-1}^{*}, g_t, \varepsilon_t, 0\right) - W\left(g_{A,t}, p_{t-1}^{*}, g_t, \varepsilon_t, 1\right).$$

We compute Δ for three models: the 'standard model' (i.e., the CGG model), our involuntary unemployment model, and our model with full information as described in section B of this appendix. We measure the impact on the welfare cost of business cycles of the assumption of imperfect insurance markets by comparing these three measures of welfare.

Our numerical strategy for approximating Δ is as follows. Perturbation methods can be used to solve the system by a second order approximation about nonstochastic steady state:

$$\varepsilon_t = 0, g_{A,t} = g_A,\ p_{t-1}^{*} = 1,\ g_t = g,\ \sigma = 0. \tag{C.3}$$

Let x_t denote one of the variables, (A.33), in the system, $x = K_p, F_p, h, p^*, \pi, R, w$. The solution to x_t is a function of the state, $g_{A,t}$, $p_{t-1}^{*}, g_t, \varepsilon_t$. In addition, it is a function of σ. The second order approximate solution is witten

$$\begin{aligned}
x\left(g_{A,t}, p_{t-1}^{*}, \varsigma_t, \sigma\right) &\simeq \hat{x}\left(g_{A,t}, p_{t-1}^{*}, g_t, \varepsilon_t, \sigma\right) \\
&\equiv x^{ss} + x_{g_A}\left(g_{A,t} - g_A\right) + x_{p^*}\left(p_{t-1}^{*} - 1\right) + x_g\left(g_t - g\right) + x_\varepsilon \varepsilon_t + x_\sigma \sigma \\
&\quad + \frac{1}{2}\left[x_{g_A,g_A}\left(g_{A,t} - g_A\right)^2 + x_{p^*,p^*}\left(p_{t-1}^{*} - 1\right)^2 + x_{g,g}\left(g_t - g\right)^2 + x_{\varepsilon\varepsilon}\varepsilon_t^2 + x_{\sigma,\sigma}\sigma^2\right] \\
&\quad + x_{g_A,p^*}\left(g_{A,t} - g_A\right)\left(p_{t-1}^{*} - 1\right) + x_{g_A,g}\left(g_{A,t} - g_A\right)\left(g_t - g\right) \\
&\quad + x_{g_A,\varepsilon}\left(g_{A,t} - g_A\right)\left(\varepsilon_t - \varepsilon\right) + x_{g_A,\sigma}\left(g_{A,t} - g_A\right)\sigma \\
&\quad + x_{p^*,g}\left(p_{t-1}^{*} - 1\right)\left(g_t - g\right) + x_{p^*,\varepsilon}\left(p_{t-1}^{*} - 1\right)\varepsilon_t + x_{p^*,\sigma}\left(p_{t-1}^{*} - 1\right)\sigma \\
&\quad + x_{g,\varepsilon}\left(g_t - g\right)\varepsilon_t + x_{g,\sigma}\left(g_t - g\right)\sigma.
\end{aligned}$$

Here, the subscripts on x denote differentiation with respect to the indicated arguments, evaluated at (C.3). Also, x^{ss} denotes the value of x in steady state, (C.3). It can be shown that $x_\sigma = x_{g_A,\sigma} = x_{p^*,\sigma} = x_{g,\sigma} = 0$. That is, uncertainty only affects the intercept terms in the second order approximation of policy rules. Slope terms are not affected. The objects in the second order approximation can be computed using the programming package, Dynare.

We define the cost of technology shocks by doing the above computations with all shock variances set to zero, except the innovation to $g_{A,t}$. Then, the welfare cost of the shocks, g_{At}, is

$$\hat{w}\left(g_{A,t}, p_{t-1}^*, \varsigma, 0\right) - \hat{w}\left(g_{A,t}, p_{t-1}^*, \varsigma, 1\right) = -\frac{w_{\sigma,\sigma}}{2}.$$

Interestingly, it is not necessary in the models we work with, for the above cost to be positive. Still, in the numerical examples we considered, $w_{\sigma,\sigma} < 0$. The above second derivative is reported by Dynare in the printout of the results when a second order approximation is requested.[42]

We need to translate a utility impact into consumption units. The present discounted value of utility in the stochastic equilibrium is:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - z\left(h_t, \varsigma\right)\right] = \hat{w}\left(g_{A,t}, p_{t-1}^*, g_t, \varepsilon_t, 1\right),$$

plus a constant term that is not dependent on the variance of the technology shock. When we shut down the variance of technology, utility is:

$$\bar{U}_0 = \sum_{t=0}^{\infty} \beta^t \left[\log \bar{C}_t - z\left(\bar{h}_t, \varsigma\right)\right] = \hat{w}\left(g_{A,t}, p_{t-1}^*, g_t, \varepsilon_t, 0\right),$$

plus a constant. The change in utility from shutting down the technology shock (say) is, approximately,

$$\Delta U = \bar{U}_0 - U_0 = \hat{w}\left(g_{A,t}, p_{t-1}^*, g_t, \varepsilon_t, 0\right) - \hat{w}\left(g_{A,t}, p_{t-1}^*, g_t, \varepsilon_t, 1\right) = -\frac{w_{\sigma,\sigma}}{2}.$$

We can convert ΔU into consumption units by asking the following. Suppose we increase consumption in the stochastic equilibrium from C_t to $(1 + \lambda/100)\, C_t$, for all t for some constant λ. What would the value of λ have to be to make utility in the stochastic equilibrium equal to utility in the nonstochastic equilibrium? In the stochastic equilibrium with adjusted consumption, utility is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - z\left(h_t, \varsigma\right)\right] + \frac{\log\left(1 + \lambda/100\right)}{1 - \beta} = U_0 + \frac{\log\left(1 + \lambda/100\right)}{1 - \beta},$$

[42] We interpret the term, '(correction)', in the Dynare printout as corresponding to $w_{\sigma,\sigma}/2$.

using the fact that logs convert multiplication into addition. We want to find λ such that

$$U_0 + \frac{\log(1+\lambda/100)}{1-\beta} = \bar{U}_0,$$

or,

$$\lambda = 100\left[e^{-(1-\beta)\frac{w_{\sigma,\sigma}}{2}} - 1\right].$$

C.3. Numerical Experiments

The results are reported in Table A2. The first column displays the model parameters in the bottom panel. Note how low the replacement ratio is, 20 percent. This is much lower than what is reported in data. Our medium sized DSGE model displays a much higher replacement ratio because the preferences in that model have habit persistence. In effect, this creates a stronger desire for insurance, and this is manifest in a much higher replacement ratio.

The second column of the table indicates the impact of information per se. The structural parameters of our model are all the same in the second column. Notice from the first panel how more people join the labor force, and the employment rate in the population, h, is higher in that model. This is not surprising. Getting people to work is costlier in the involuntary unemployment model because a premium has to be offered to people that are lucky to find work, and this in effect reduces insurance. In the full information model, no insurance sacrifice is required to get people to work. In that model, the family issues instructions to each household that are contingent on that household's aversion to work. No incentives are required to enforce those instructions, and so the family can allocate consumption equally across families. We calculate the price of information by asking what constant percent increase in consumption (analogous to λ in the previous section) would make households in the involuntary unemployment model indifferent between remaining in that world and going to the full information world. The price is 0.2 percent of consumption.

The third and fourth columns represents our attempt to address the Lucas conjecture. Those columns address the question, "what would an econometrician living in our involuntary unemployment world, who incorrectly assumes insurance markets are perfect, conclude about the welfare cost of business cycles?". The correct approach to this question, in our view, is to make a plausible assumption about the data available to the econometrician, e.g., output, consumption, government consumption, inflation, the interest rate generated by the model in column 1 of Table A2. A reasonable first step would avoid sampling uncertainty by supposing that the econometrician has a large data set. Then, one identifies the parameter values that the econometrician who does Gaussian maximum likelihood would estimate (the Gaussian likelihood function is easy to characterize when there is a large sample of data by expressing it in the frequency domain). We did not do this computation. We did an alternative computation instead, and we describe this when we describe our experiments in detail below.

In addition to deciding how the econometrician comes up with parameter values, one has to determine exactly what model the econometrician works with. Our first approach assumes that the econometrician uses the full information model described in section B. That model, relative to the model in the first column has precisely one specification error. It makes the mistake of supposing the family has all information on households, whereas in fact the family only knows the household's employment status. A second approach is to imagine the econometrician uses what we call the standard model, the CGG model. This in effect makes two specification errors. It assumes effort is not required to find a job and although households still face risk in terms of their aversion to work, they have full consumption insurance against this risk. The results in column three ('Full information model, observational equivalent') refer to the first approach and the results in the next column refer to the second. The first approach has the appealing feature that it isolates clearly the nature of the specification error being commited. The second approach has the appealing feature that it works with the standard, simple utility function used in practice in the analysis of DSGE models.

C.3.1. The Full Information Model

For our computation, we assumed that the data are generated by the model in column 1 of Table A2. The model used by the econometrician is the full information model. We imagine that the econometrician is provided with m and h, the steady state values of the labor force and hours worked, respectively. Of course, a trivial identify then permits him to compute the steady state unemployment rate. The econometrician is also provided with the parameter values reported in Table A1. Regarding the Taylor rule, we may perhaps suppose that he has data on the variables in that equation (including the output gap!) and he runs an instrumental variables regression on the policy rule, using the lagged gap, inflation and interest rates as instruments (recall, the policy shock is in fact iid, and the econometrician knows it). Regarding the price stickiness parameter, we may interpret the econometrician's knowing the value of this parameter as reflecting that he read a micro study about price stickiness. In the case of λ_f, he may have gotten this parameter from an Industrial Organization colleague. The steady state of technology growth can be inferred from the average growth rate of output in the economy. The steady state government consumption to GDP ratio could have been computed from a time series average of that variable.

The econometrician is also provided with the values of κ^{Okun} and σ_z. We may suppose that ordinary least squares using data on the inflation rate and the output gap is used to estimate the slope of the Phillips curve, (2.45). Then, σ_z can be unravelled from the slope given the known values of ξ_p, β and η_g. Similarly, κ^{Okun} may be found by computing the ratio of the output gap and the unemployment gap in any date. It turns out that this is not enough for the econometrician to solve the model. For this, he needs to determine values of the five parameters in (C.1). We need to provide him with one additional piece of information. We provide the econometrician with $\bar{p}$, the steady state probability of finding a job by the household who has the least aversoin to work, $l = 0$. The fact that we must provide $\bar{p}$ indicates that, of the data we have already provided, there is not enough to determine the values of the parameters in (C.1).

Given this strategy attributed to the econometrician for parameterizing his misspecified model, the results are given in the third column of table A2. Note how the results in the first panel are the same for the two models (except, of course, for the replacement ratio). The structual parameters, (C.1), estimated by the econometrician (see the middle panel) are of course different from their correct values, the ones reported in the first column. The inconsistency of the econometrician's estimator reflects the specification error he commits when he assumes that the family has full information. The inconsistency in two parameters, η and σ_L, is so small it is not evident given the number of significant digits reported in the table. The inconsistency of the econometrician's estimator of the three other parameters, however, is substantial.

The bottom panel reports the welfare cost of business cycles computed by the econometrician using his estimated model. This is to be compared with the true welfare cost of inflation, reported in the first column. The notable result here is that the welfare costs are all identical to many significant digits, regarless of the shocks considered. This result was a surprise to us. It is true that the econometrician estimates, relative to the data provided to him, a model that is observationaly equivalent to the true model. But, this estimation is presumed to be done using the first order properties of the model. Because labor supply is part of the system (and this involves the first derivative of the utility function), this means that the econometrician using first order expansions chooses parameters to match the second derivative of the utility function. But, welfare is computed by taking a second order expansion of the system, and so one presumes that it involves the third derivative of the utility function. That is, our welfare computation focuses on a feature of the utility function (i.e., its third derivative) that the econometrician has no reason to match to the corresponding true value in the process of estimation. This is why we expected the welfare cost of business cycles to be different between the models. The fact that the numbers are numerically so extremely close suggests to us that there exists a theorem that declares the numbers are mathematically the same. We have not developed a proof for that theorem. From this perspective, it is interesting to note that when we perturbed the computations in particular ways, the actual magnitude of the welfare losses did not change. In particular, we perturbed the parameters of the data generating mechanism, i.e., the model in the first column, so as to change the values of the endogenous variables provided to the econometrician. In these experiments, we only considered the case of neutral technology shocks (we suppose that the results would have been had we worked with the other shocks). We redid the computations holding $\bar{p}$, h, m, σ_z fixed at their values in Table A2 and changed the value of σ_L (thereby implicitly changing κ^{Okun}) to 4. We then changed the value of $\bar{p}$ to 0.97, holding h, m, σ_z, σ_L fixed at their values reported in the table. We changed the value of u to 0.07, holding $\bar{p}$, m, σ_z, σ_L to their values in the table. In all these cases, the magnitude of the welfare cost of business cycles remained unchanged. Only when we changed σ_z (to unity) did the welfare cost of business cycles change. However, even in this case, the econometrician's estimate of the welfare cost of business cycles is the same, to 10 digits, as the true cost.[43]

The computations in the third column of the table are consistent with the proposition that abstracting from incomplete insurance does not distort assessments about the welfare cost of business cycles.

C.3.2. The Standard Model

Consider now the fourth column. This column presents results under the assumption that the econometrician's model corresponds to the standard model, the one in which the family utility function is given by (3.21) with $b = 0$. Again, we must specify what the econometrician does to parameterize his misspecified model. As before, we assume that the econometrician knows the values of the parameters in Table A1. Now, however, the econometrician has only to estimate two parameters of the utility function, the curvature and slope parameters, σ_L, and ς, respectively.

We suppose that the econometrician knows the slope of the Phillips curve and can therefore recover σ_z. In the econometician's model the value of σ_L coincides with that of σ_z. The econometrician then estimates ς by requiring that steady state hours worked in his model coincide with the actual steady state of hours worked. We suppose that the econometrician simply ignores unemployment and labor force data, as in standard monetary DSGE models.

The results are given in the fourth column of table A2. The key finding in Table A2 is that the welfare cost of business cycles estimated by the econometrician coincides with the true welfare cost, up to 12 significant digits!

[43] In this case, the welfare cost of business cycles is 0.553167318878733 for column 1 and 0.553167318879688 for column 3.

D. Non-Separability in Utility

In the manuscript, we work with a household utility function in which consumption and leisure are additively separable. In this appendix, we show that the analysis can also easily be done with two non-separable utility functions that have been used extensively in the literature.

D.1. King-Plosser-Rebelo Preferences

We replace the preferences in (2.4) with:

$$p(e_t)\frac{(c_t^w)^{1-\gamma}}{1-\gamma}v(l) + (1-p(e_t))\frac{(c_t^{nw})^{1-\gamma}}{1-\gamma}v(0) - \frac{1}{2}\frac{e_t^2}{1-\gamma},\ \gamma\ >\ 1$$
$$v(l) = F + \varsigma_t(1+\sigma_L)l^{\sigma_L}.$$

With these preferences, utility is decreasing in l, and the marginal utility of consumption is increasing in l. In this case, the household with aversion to work, l, sets

$$e_{l,t} = \max\left\{a\left[(c_t^w)^{1-\gamma}v(l) - (c_t^{nw})^{1-\gamma}v(0)\right], 0\right\}.$$

In this case, households that participate in the labor market receive utility

$$\frac{1}{2}a^2\left[\frac{(c_t^w)^{1-\gamma}}{1-\gamma}v(l) - \frac{(c_t^{nw})^{1-\gamma}}{1-\gamma}v(0)\right]^2 + \frac{(c_t^{nw})^{1-\gamma}}{1-\gamma}v(0),$$

while households that do not participate receive

$$\frac{(c_t^{nw})^{1-\gamma}}{1-\gamma}v(0).$$

The incentive constraint (i.e., the analog of (2.9)) requires:

$$(c_t^w)^{1-\gamma}v(m) = (c_t^{nw})^{1-\gamma}v(0) \tag{D.1}$$

The mapping from the labor force, m_t, to the number of people working, h_t, is given by:

$$h_t = \int_0^{m_t} p(e_{l,t})\,dl = a^2\frac{(c_t^w)^{1-\gamma}}{\gamma-1}\varsigma_t(1+\sigma_L)\int_0^{m_t}\left[m_t^{\sigma_L} - l^{\sigma_L}\right]dl = a^2\frac{(c_t^w)^{1-\gamma}}{\gamma-1}\varsigma_t\sigma_L m_t^{\sigma_L+1}. \tag{D.2}$$

Combining the resource constraint, (2.18), with the incentive constraint, (D.1), we obtain:

$$c_t^w = \frac{C_t}{h_t + (1-h_t)\left[\frac{v(m_t)}{v(0)}\right]^{\frac{1}{1-\gamma}}} \tag{D.3}$$

$$c_t^{nw} = \frac{C_t\left[\frac{v(m_t)}{v(0)}\right]^{\frac{1}{1-\gamma}}}{h_t + (1-h_t)\left[\frac{v(m_t)}{v(0)}\right]^{\frac{1}{1-\gamma}}}. \tag{D.4}$$

Integrating utility over all the households in the family, the analog of (2.20) is:

$$u\left(c_t^w, c_t^{nw}, m_t\right) = a^2 \frac{\left(c_t^w\right)^{2(1-\gamma)}}{(1-\gamma)^2}\left(1+\sigma_L\right)\frac{\sigma_L^2 m_t^{2\sigma_L+1}}{\left(2\sigma_L+1\right)} + \frac{\left(c_t^{nw}\right)^{1-\gamma}}{1-\gamma} v\left(0\right). \tag{D.5}$$

Equations (D.2), (D.3) and (D.4) provide a mapping from C_t and h_t to c_t^w, c_t^{nw} and m_t. Utility is then given by (D.5). Thus, we have family utility in terms of C_t and h_t only.

D.2. Greenwood-Hercowitz-Huffman Preferences

We now replace the preferences in (2.4) with:

$$p\left(e_t\right)\frac{\left(c_t^w + F - \left(1+\sigma_L\right) l^{1+\sigma_L}\right)^{1-\gamma} - 1}{1-\gamma} + \left(1 - p\left(e_t\right)\right)\frac{\left(c_t^{nw} + F\right)^{1-\gamma} - 1}{1-\gamma} - \frac{1}{2}e_t^2, \ \gamma > 0.$$

In this case,

$$e_{l,t} = \max\left\{ a\left[\frac{\left(c_t^w + F - \left(1+\sigma_L\right) l^{1+\sigma_L}\right)^{1-\gamma} - \left(c_t^{nw} + F\right)^{1-\gamma}}{1-\gamma}\right], 0\right\},$$

so that the utility of a household that participates in the labor force is:

$$\frac{1}{2}a^2\left[\frac{\left(c_t^w + F - \left(1+\sigma_L\right) l^{1+\sigma_L}\right)^{1-\gamma} - \left(c_t^{nw} + F\right)^{1-\gamma}}{1-\gamma}\right]^2 + \frac{\left(c_t^{nw} + F\right)^{1-\gamma} - 1}{1-\gamma}.$$

Comparing this with the utility of households that choose to be out of the labor force, we obtain the incentive constraint:

$$c_t^w - \left(1+\sigma_L\right) m_t^{1+\sigma_L} = c_t^{nw} \tag{D.6}$$

The mapping between the labor force and the number of people working is provided by:

$$h_t = \int_0^{m_t} p\left(e_{l,t}\right) dl = \frac{a^2}{1-\gamma}\int_0^{m_t}\left[\begin{array}{c}\left(c_t^w + F - \left(1+\sigma_L\right) l^{1+\sigma_L}\right)^{1-\gamma} \\ -\left(c_t^w + F - \left(1+\sigma_L\right) m_t^{1+\sigma_L}\right)^{1-\gamma}\end{array}\right] dl. \tag{D.7}$$

Now,

$$\begin{aligned} & \int_0^{m_t}\left[c_t^w + F - \left(1+\sigma_L\right) l^{1+\sigma_L}\right]^{1-\gamma} dl \\ = \ & m_t\left[c_t^w + F - \left(1+\sigma_L\right) m_t^{1+\sigma_L}\right]^{1-\gamma}\left(\frac{c_t^w + F - \left(1+\sigma_L\right) m_t^{(1+\sigma_L)}}{c_t^w + F}\right)^{-(1-\gamma)} \\ & \times\mathcal{F}\left(\left[\ -(1-\gamma) \quad \frac{1}{1+\sigma_L}\ \right]; 1 + \frac{1}{1+\sigma_L}; \frac{\left(1+\sigma_L\right) m_t^{1+\sigma_L}}{c_t^w + F}\right), \end{aligned}$$

where

$$\mathcal{F}\left(x; a; b\right)$$

denotes the hypergeometric function where x is a 1×2 row vector and a and b are scalars.[44] In this way, (D.7) defines a mapping from c_t^w, c_t^{nw} and m_t to h_t :

$$h_t = f\left(c_t^w, c_t^{nw}, m_t\right). \tag{D.8}$$

Combining the resource constraint, (2.18), with the incentive constraint, (D.6), we obtain:

$$c_t^w = C_t + (1 - h_t)(1 + \sigma_L) m_t^{1+\sigma_L} \tag{D.9}$$

$$c_t^{nw} = C_t - h_t (1 + \sigma_L) m_t^{1+\sigma_L} \tag{D.10}$$

Ex ante utility of all the households in the family, the analog of (2.20), is (after using the incentive constraint):

$$\begin{aligned} u\left(c_t^w, c_t^{nw}, m_t\right) &= \frac{1}{2}\frac{a^2}{(1-\gamma)^2}\int_0^{m_t}\left[\begin{array}{c}\left(c_t^w + F - (1+\sigma_L) l^{1+\sigma_L}\right)^{1-\gamma} \\ -\left(c_t^{nw} + F\right)^{1-\gamma}\end{array}\right]^2 dl \\ &+ \frac{\left(c_t^{nw} + F\right)^{1-\gamma} - 1}{1-\gamma} \end{aligned} \tag{D.11}$$

Expanding the square term:

$$\begin{aligned} &\int_0^{m_t}\left[\left(c_t^w + F - (1+\sigma_L) l^{1+\sigma_L}\right)^{1-\gamma} - \left(c_t^{nw} + F\right)^{1-\gamma}\right]^2 dl \\ = &\int_0^{m_t}\left(c_t^w + F - (1+\sigma_L) l^{1+\sigma_L}\right)^{2(1-\gamma)} dl \\ &-2\left(c_t^{nw} + F\right)^{1-\gamma}\int_0^{m_t}\left(c_t^w + F - (1+\sigma_L) l^{1+\sigma_L}\right)^{2(1-\gamma)} dl \\ &+ m_t\left(c_t^{nw} + F\right)^{2(1-\gamma)}, \end{aligned}$$

which can be evaluated using formulas analogous to the one after (D.7). Equations (D.8), (D.9) and (D.10) provide a mapping from C_t and h_t to c_t^w, c_t^{nw} and m_t. Utility is then given by (D.11). Thus, we have family utility in terms of C_t and h_t only.

E. Solving the Model Used in the Empirical Analysis

We first derive the equilibrium conditions associated with optimal wage setting. We then indicate the remaining equilibrium conditions of the model. Finally, we describe a strategy for solving the model's steady state.

[44] This formula may be found at http://integrals.wolfram.com/index.jsp. In MATLAB, the hypergeometric function is evaluated using hypergeom(x,a,b).

E.1. Scaling of Variables

We adopt the following scaling of variables. The neutral shock to technology is z_t and its growth rate is $\mu_{z,t}$:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$

The variable, Ψ_t, is an embodied shock to technology and it is convenient to define the following combination of embodied and neutral technology:

$$\begin{aligned} z_t^+ &\equiv \Psi_t^{\frac{\alpha}{1-\alpha}} z_t, \\ \mu_{z^+,t} &\equiv \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}. \end{aligned} \tag{E.1}$$

Capital, $\bar{K}_t$, and investment, I_t, are scaled by $z_t^+ \Psi_t$. Consumption goods C_t, government consumption G_t and the real wage, W_t/P_t are scaled by z_t^+. Also, v_t is the multiplier on the nominal household budget constraint in the Lagrangian version of the household problem. That is, v_t is the marginal utility of one unit of currency. The marginal utility of a unit of consumption is $v_t P_t$. The latter must be multiplied by z_t^+ to induce stationarity. Thus, our scaled variables are:

$$\begin{aligned} k_{t+1} &= \frac{K_{t+1}}{z_t^+ \Psi_t},\ \bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_t^+ \Psi_t},\ i_t = \frac{I_t}{z_t^+ \Psi_t},\ c_t = \frac{C_t}{z_t^+},\ g_t = \frac{G_t}{z_t^+},\ \bar{w}_t = \frac{W_t}{z_t^+ P_t} \\ \psi_{z^+,t} &= v_t P_t z_t^+,\ \tilde{y}_t = \frac{Y_t}{z_t^+},\ \tilde{p}_t = \frac{\tilde{P}_t}{P_t},\ w_t = \frac{\tilde{W}_t}{W_t},\ \dddot{w}_t \equiv \frac{\dddot{W}_t}{W_t},\ \overset{\infty}{w}_t = \frac{\overset{\infty}{W}_t}{W_t}. \end{aligned} \tag{E.2}$$

We define the scaled date t price of new installed physical capital for the start of period $t+1$ as $p_{k',t}$ and we define the scaled real rental rate of capital as $\bar{r}_t^k$:

$$p_{k',t} = \Psi_t P_{k',t},\ \bar{r}_t^k = \Psi_t r_t^k.$$

where $P_{k',t}$ is in units of the homogeneous good. We define the following inflation rates:

$$\pi_t = \frac{P_t}{P_{t-1}},\ \pi_t^i = \frac{P_t^i}{P_{t-1}^i}.$$

Here, P_t is the price of the homogeneous output good and P_t^i is the price of the domestic final investment good.

E.2. Wage Setting by the Family

We consider the problem of a monopolist who represents households that supply the type j labor service. That monopolist optimizes the utility function of $j-$type households, (2.21), subject to Calvo frictions. With probability $1 - \xi_w$ the monopolist reoptimizes the wage and with probability ξ_w the monopolist sets the current wage rate according to (3.10). In each period, type j households supply the quantity of labor dictated by demand, (3.6). Because the $j-$type family has perfect consumption insurance, the monopolist can take the $j-$type family's consumption as given. However, the monopolist does assign a weight to the revenues from $j-$type labor that corresponds to the value, υ_t, assigned to income by the family. Ignoring terms beyond the control of the monopolist the monopolist seeks to maximize:

$$E_t^j \sum_{i=0}^{\infty} \beta^i \left[-z\left(h_{t+i,j}, \zeta_{t+i}\right) + \upsilon_{t+i} W_{t+i,j} h_{t+i,j} \right].$$

Here, υ_t denotes the Lagrange multiplier on the type j family's time t flow budget constraint, (2.2). The function, z, is defined in (2.21), but we reproduce it here for convenience:

$$z\left(h_{j,t}, \zeta_t\right) = \log\left[h_{j,t}\left(e^{F+\varsigma(1+\sigma_L) f(h_t)^{\sigma_L}} - 1\right) + 1\right] - \frac{a^2 \varsigma^2 \left(1+\sigma_L\right)\sigma_L^2}{2\sigma_L + 1} f\left(h_{j,t}\right)^{2\sigma_L+1} - \eta\varsigma\sigma_L f\left(h_{j,t}\right)^{\sigma_L+1}.$$

Here, $f\left(h_{j,t}, \zeta_t\right)$ is the unique value of m_t that satisfies

$$h_{j,t} = Q\left(m_{j,t}, \zeta_t\right) \equiv m_{j,t}\eta + a^2 \varsigma_t \sigma_L m_{j,t}^{\sigma_L+1}, \tag{E.3}$$

for a given value of m_t. That is,

$$m_{j,t} = f\left(h_{j,t}, \zeta_t\right) \equiv Q^{-1}\left(h_{j,t}, \zeta_t\right),$$

where Q^{-1} is the inverse function of Q. In the case of the standard model, z is implicitly defined in (3.21).

E.2.1. Differentiating the Family Disutility of Labor

In the calculations that follow, we require the derivatives of z and f, evaluated in steady state. In the case of the standard model, these calculations are trivial. We compute the derivatives for our model with involuntary unemployment here. We drop the j subscript for convenience, as well as the stochastic shock. From the definition of the inverse function,

$$m_t = f\left(Q\left(m_t\right)\right).$$

We find the derivatives of f by differentiating this expression twice with respect to m_t :

$$\begin{aligned} 1 &= f'\left(Q\left(m_t\right)\right) Q'\left(m_t\right) \\ 0 &= f''\left(Q\left(m_t\right)\right)\left[Q'\left(m_t\right)\right]^2 + f'\left(Q\left(m_t\right)\right) Q''\left(m_t\right) \end{aligned}$$

From the first expression,

$$f'\left(Q\left(m_t\right)\right) = \frac{1}{Q'\left(m_t\right)}.$$

Substituting this into the second expression and solving:

$$0 = f''\left(Q\left(m_t\right)\right)\left[Q'\left(m_t\right)\right]^2 + \frac{Q''\left(m_t\right)}{Q'\left(m_t\right)},$$

so that

$$f''\left(Q\left(m_t\right)\right) = -\frac{Q''\left(m_t\right)}{\left[Q'\left(m_t\right)\right]^3},$$

From (E.3),

$$\begin{aligned} Q'\left(m_t\right) &= \eta + \left(\sigma_L + 1\right) a^2 \varsigma \sigma_L m_t^{\sigma_L} \\ Q''\left(m_t\right) &= \left(\sigma_L + 1\right) a^2 \varsigma \sigma_L^2 m_t^{\sigma_L - 1}, \end{aligned}$$

so that, in steady state,

$$\begin{aligned} f_h &= \frac{1}{\eta + \left(\sigma_L + 1\right) a^2 \varsigma \sigma_L m^{\sigma_L}} \\ f_{hh} &= -\frac{\left(\sigma_L + 1\right) a^2 \varsigma \sigma_L^2 m^{\sigma_L - 1}}{\left[\eta + \left(\sigma_L + 1\right) a^2 \varsigma \sigma_L m^{\sigma_L}\right]^3}, \end{aligned} \tag{E.4}$$

where m denotes the steady state value of m, computed below.

Let,

$$\begin{aligned} Z\left(m_t\right) &= \log\left[Q\left(m_t\right)\left(e^{F+\varsigma\left(1+\sigma_L\right) m_t^{\sigma_L}} - 1\right) + 1\right] - \frac{a^2 \varsigma^2 \left(1 + \sigma_L\right) \sigma_L^2}{2\sigma_L + 1} m_t^{2\sigma_L + 1} - \eta \varsigma \sigma_L m_t^{\sigma_L + 1} \\ h_t &= Q\left(m_t\right) \equiv m_t \eta + a^2 \varsigma \sigma_L m_t^{\sigma_L + 1}, \end{aligned}$$

so that

$$z\left(h_t\right) \equiv Z\left(Q^{-1}\left(h_t\right)\right) = Z\left(f\left(h_t\right)\right),$$

and

$$\begin{aligned} z_h\left(h_t\right) &= Z'\left(f\left(h_t\right)\right) f_h\left(h_t\right) \\ z_{hh}\left(h_t\right) &= Z''\left(f\left(h_t\right)\right)\left[f_h\left(h_t\right)\right]^2 + Z'\left(f\left(h_t\right)\right) f_{hh}\left(h_t\right). \end{aligned}$$

Evaluating this in steady state,

$$z_h = Z' f_h, \;\; z_{hh} = Z'' f_h^2 + Z' f_{hh}.$$

In this case,

$$\sigma_z \equiv \frac{z_{hh} h}{z_h} = \frac{\left[Z'' f_h^2 + Z' f_{hh}\right] h}{Z' f_h} = \frac{Z'' f_h h}{Z'} + \frac{f_{hh} h}{f_h}$$

From (E.4),

$$f_h = \frac{1}{Q'}, \;\; f_{hh} = -\frac{Q''}{\left[Q'\right]^3},$$

so that

$$\begin{aligned}\sigma_z &= \frac{Z'' f_h h}{Z'} + \frac{f_{hh} h}{f_h} \\ &= \frac{Z'' h}{Z' Q'} - \frac{Q'' h}{[Q']^2} \\ &= \frac{Q}{Q'}\left[\frac{Z''}{Z'} - \frac{Q''}{Q'}\right]\end{aligned}$$

E.2.2. First Order Condition Associated with Family Wage Setting

Consider the monopoly wage setter, j, that has an opportunity to reoptimize the wage rate. The objective function with $h_{t+i,j}$ substituted out using labor demand, (3.6), and ignoring terms beyond the control of the monopolist, is as follows:

$$E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i [-z\left(\zeta_{t+i}, \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)$$

$$+\upsilon_{t+i} \tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}],$$

where

$$\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}$$

is the nominal wage rate of the monopolist which sets wage $\tilde{W}_t$ in period t and cannot reoptimize again afterward. Also, z is the function described in the case of our model with unemployment, and it is (3.21) in the case of the standard model. We adopt the following scaling convention:

$$w_t = \frac{\tilde{W}_t}{W_t},\ \bar{w}_t = \frac{W_t}{z_t^+ P_t},\ \psi_{z^+,t} = \upsilon_t P_t z_t^+.$$

With this notation, the objective can be written,

$$E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i [-z\left(\zeta_{t+i}, \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right) + \psi_{z^+,t+i} w_t^{\frac{1}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}],$$

where:

$$X_{t,i} = \frac{\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{\pi_{t+i}\pi_{t+i-1} \cdots \pi_{t+1}\mu_{z^+,t+i} \cdots \mu_{z^+,t+1}}.$$

Differentiating with respect to w_t,

$$E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i [-z_{h,t+i}^t \frac{\lambda_w}{1-\lambda_w} w_t^{\frac{\lambda_w}{1-\lambda_w}-1} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}$$

$$+\frac{1}{1-\lambda_w} \psi_{z^+,t+i} w_t^{\frac{1}{1-\lambda_w}-1} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}],$$

where

$$z_{h,t+i}^{t} \equiv z_h\left(\zeta_{t+i}, \left(\frac{w_t\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right).$$

Here, $h_{2,t+i}^{t}$ denotes the marginal utility of labor in period $t+i$, for a monopolist who last reoptimized the wage rate in period t. Dividing and rearranging,

$$E_t\sum_{i=0}^{\infty}(\beta\xi_w)^i\left(\frac{\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}[\psi_{z^+,t+i}w_t\bar{w}_tX_{t,i} - \lambda_w z_{h,t+i}^{t}] = 0. \tag{E.5}$$

The first object in square brackets is the marginal utility real wage in period $t+i$ and the second is a markup, λ_w, over the marginal utility cost of working. According to (E.5) the monopolist attempts to set a weighted average of the term in square brackets to zero. The structure of $z_{z,t+i}^{t}$ makes it difficult to express (E.5) in recursive form. This is because we have not found a way to express $z_{h,t+1}^{t} = Z_t z_{h,t+1}^{t+1}$, for some variable, Z_t. The expression, (E.5), is recursive after linearizing it about steady state. Thus,

$$\hat{z}_{h,t+i}^{t} \equiv \frac{dz_h\left(\zeta_{t+i}, \left(\frac{w_t\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)}{z_h\left(\zeta, w^{\frac{\lambda_w}{1-\lambda_w}}H\right)},$$

where a variable without a time subscript denotes non-stochastic steady state. Expanding this expression:

$$\hat{z}_{h,t+i}^{t} = \sigma_\varsigma\hat{\zeta}_{t+i} + \alpha_{h,1}\left(\hat{w}_t + \hat{\bar{w}}_t - \hat{\bar{w}}_{t+i} + \hat{X}_{t,i}\right) + \sigma_z\hat{H}_{t+i},$$

where,

$$\sigma_\varsigma \equiv \frac{z_{h\zeta}\zeta}{z_h},\ \ \sigma_z \equiv \frac{z_{hh}H}{z_h},\ \ \alpha_{h,1} \equiv \frac{\lambda_w}{1-\lambda_w}\sigma_z.$$

Also,

$$\hat{X}_{t,i} = \hat{\bar{\pi}}_{w,t+i} + \cdots + \hat{\bar{\pi}}_{w,t+1} - \hat{\pi}_{t+i} - \hat{\pi}_{t+i-1} - \cdots - \hat{\pi}_{t+1} - \hat{\mu}_{z^+,t+i} - \cdots - \hat{\mu}_{z^+,t+1}.$$

However, note:

$$\hat{\bar{\pi}}_{w,t+1} = \kappa_w\hat{\pi}_t.$$

Then,

$$\hat{X}_{t,i} = -\Delta_{\kappa_w}\hat{\pi}_{t+i} - \Delta_{\kappa_w}\hat{\pi}_{t+i-1} - \cdots - \Delta_{\kappa_w}\hat{\pi}_{t+1} - \hat{\mu}_{z^+,t+i} - \cdots - \hat{\mu}_{z^+,t+1},$$

where

$$\Delta_{\kappa_w} \equiv 1 - \kappa_w L,$$

where L denotes the lag operator.

Write out (E.5) in detail:

$$\begin{aligned}
&H_t[\psi_{z^+,t} w_t \bar{w}_t - \lambda_w z^t_{h,t}] \\
&+\beta\xi_w \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} X_{t,1}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+1}[\psi_{z^+,t+1} w_t \bar{w}_t X_{t,1} - \lambda_w z^t_{h,t+1}] \\
&+ (\beta\xi_w)^2 \left(\frac{\bar{w}_t}{\bar{w}_{t+2}} X_{t,2}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2}[\psi_{z^+,t+2} w_t \bar{w}_t X_{t,2} - \lambda_w z^t_{h,t+2}] + ... = 0
\end{aligned}$$

In expanding this expression, we can simply set the terms outside the square brackets to their steady state values. The reason is that the term inside the brackets are equal to zero in steady state. Thus, the expansion of the previous expression about steady state:

$$\begin{aligned}
&H[d\left(\psi_{z^+,t} w_t \bar{w}_t\right) - \lambda_w d\left(z^t_{h,t}\right)] \\
&+\beta\xi_w H[d\left(\psi_{z^+,t+1} w_t \bar{w}_t X_{t,1}\right) - \lambda_w d\left(z^t_{h,t+1}\right)] \\
&+ (\beta\xi_w)^2 H[d\left(\psi_{z^+,t+2} w_t \bar{w}_t X_{t,2}\right) - \lambda_w d\left(z^t_{h,t+2}\right)] + ... = 0
\end{aligned}$$

or,

$$\begin{aligned}
&H[\psi_{z^+}\bar{w}\left(\hat{\psi}_{z^+,t} + \hat{w}_t + \widehat{\bar{w}}_t\right) - \lambda_w z_h \hat{z}^t_{h,t}] \\
&+\beta\xi_w H[\psi_{z^+}\bar{w}\left(\hat{\psi}_{z^+,t+1} + \hat{w}_t + \widehat{\bar{w}}_t + \hat{X}_{t,1}\right) - \lambda_w z_h \hat{z}^t_{h,t+1}] \\
&+ (\beta\xi_w)^2 H[\psi_{z^+}\bar{w}\left(\hat{\psi}_{z^+,t+2} + \hat{w}_t + \widehat{\bar{w}}_t + \hat{X}_{t,2}\right) - \lambda_w z_h \hat{z}^t_{h,t+2}] + ... = 0
\end{aligned}$$

Note that in steady state, $\psi_{z^+}\bar{w} = \lambda_w z_h$, so that, after multiplying by $1/\left(H\psi_{z^+}\bar{w}\right)$, we obtain:

$$\begin{aligned}
&\hat{\psi}_{z^+,t} + \hat{w}_t + \widehat{\bar{w}}_t - \hat{z}^t_{h,t} \\
&+\beta\xi_w[\hat{\psi}_{z^+,t+1} + \hat{w}_t + \widehat{\bar{w}}_t + \hat{X}_{t,1} - \hat{z}^t_{h,t+1}] \\
&+ (\beta\xi_w)^2 [\hat{\psi}_{z^+,t+2} + \hat{w}_t + \widehat{\bar{w}}_t + \hat{X}_{t,2} - \hat{z}^t_{h,t+2}] + ... = 0
\end{aligned}$$

Substitute out for $\hat{z}^t_{h,t+i}$ and $\hat{X}_{t,i}$:

$$\begin{aligned}
0 \;=\; &\hat{\psi}_{z^+,t} + \hat{w}_t + \widehat{\bar{w}}_t - \left[\sigma_\varsigma \hat{\zeta}_t + \alpha_{h,1}\hat{w}_t + \sigma_z \hat{H}_t\right] \\
&+\beta\xi_w[\hat{\psi}_{z^+,t+1} + \hat{w}_t + \widehat{\bar{w}}_t - \left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right) \\
&- \left(\sigma_\varsigma \hat{\zeta}_{t+1} + \alpha_{h,1}\left(\hat{w}_t + \widehat{\bar{w}}_t - \widehat{\bar{w}}_{t+1} - \left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right)\right) + \sigma_z \hat{H}_{t+1}\right)] \\
&+ (\beta\xi_w)^2 [\hat{\psi}_{z^+,t+2} + \hat{w}_t + \widehat{\bar{w}}_t - \left(\Delta_{\kappa_w}\hat{\pi}_{t+2} + \hat{\mu}_{z^+,t+2}\right) - \left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right) \\
&- \left(\sigma_\varsigma \hat{\zeta}_{t+2} + \alpha_{h,1}\left(\begin{array}{c} \hat{w}_t + \widehat{\bar{w}}_t - \widehat{\bar{w}}_{t+2} \\ - \left(\Delta_{\kappa_w}\hat{\pi}_{t+2} + \hat{\mu}_{z^+,t+2}\right) - \left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right) \end{array}\right) + \sigma_z \hat{H}_{t+2}\right)] + ...
\end{aligned}$$

Collecting terms:

$$\begin{aligned}
0 &= \sum_{j=0}^{\infty} (\beta\xi_w)^j \left[\hat{\psi}_{z^+,t+j} - \left(\sigma_\varsigma \hat{\zeta}_{t+j} + \sigma_z \hat{H}_{t+j}\right)\right] + \frac{1-\alpha_{h,1}}{1-\beta\xi_w}\hat{w}_t \\
&+ \frac{1-\alpha_{h,1}\beta\xi_w}{1-\beta\xi_w}\widehat{\bar{w}}_t + \alpha_{h,1}\sum_{j=1}^{\infty}(\beta\xi_w)^j\,\widehat{\bar{w}}_{t+j} \\
&- (1-\alpha_{h,1})\,\beta\xi_w\left[\left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right)\right] \\
&- (1-\alpha_{h,1})\,(\beta\xi_w)^2\left[\left(\Delta_{\kappa_w}\hat{\pi}_{t+2} + \hat{\mu}_{z^+,t+2}\right) + \left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right)\right] \\
&- ...
\end{aligned}$$

or,

$$0 = \sum_{j=0}^{\infty} (\beta\xi_w)^j \left[\hat{\psi}_{z^+,t+j} - \left(\sigma_\varsigma \hat{\zeta}_{t+j} + \sigma_z \hat{H}_{t+j}\right)\right] + \frac{1-\alpha_{h,1}}{1-\beta\xi_w}\hat{w}_t$$
$$+\frac{1-\alpha_{h,1}\beta\xi_w}{1-\beta\xi_w}\widehat{\bar{w}}_t + \sum_{j=1}^{\infty}(\beta\xi_w)^j\left[\alpha_{h,1}\widehat{\bar{w}}_{t+j} - \frac{1-\alpha_{h,1}}{1-\beta\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{t+j} + \hat{\mu}_{z^+,t+j}\right)\right].$$

Note

$$\begin{aligned}
S_t &= X_t + \beta\xi_w X_{t+1} + (\beta\xi_w)^2 X_{t+2} + ... \\
&= X_t + \beta\xi_w \overbrace{[X_{t+1} + \beta\xi_w X_{t+2} + ...]}^{S_{t+1}},
\end{aligned}$$

so that the log-linearized first order condition can be written:

$$0 = F_{,t} + \frac{1-\alpha_{h,1}}{1-\beta\xi_w}\hat{w}_t + \frac{1-\alpha_{h,1}\beta\xi_w}{1-\beta\xi_w}\widehat{\bar{w}} + G_t, \tag{E.6}$$

where

$$\begin{aligned}
F_t &= \sum_{j=0}^{\infty} (\beta\xi_w)^j \left[\hat{\psi}_{z^+,t+j} - \left(\sigma_\varsigma \hat{\zeta}_{t+j} + \sigma_z \hat{H}_{t+j}\right)\right] \\
&= \hat{\psi}_{z^+,t} - \left(\sigma_\varsigma \hat{\zeta}_t + \sigma_z \hat{H}_t\right) + \beta\xi_w F_{t+1} \\
G_t &= \sum_{j=1}^{\infty}(\beta\xi_w)^j\left[\alpha_{h,1}\widehat{\bar{w}}_{t+j} - \frac{1-\alpha_{h,1}}{1-\beta\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{t+j} + \hat{\mu}_{z^+,t+j}\right)\right] \\
&= \beta\xi_w\alpha_{h,1}\widehat{\bar{w}}_{t+1} - \frac{(1-\alpha_{h,1})\,\beta\xi_w}{1-\beta\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right) + \beta\xi_w G_{t+1}
\end{aligned}$$

Note:

$$\left(1-\beta\xi_w L^{-1}\right)F_t \equiv F_t - \beta\xi_w F_{t+1} = \hat{\psi}_{z^+,t} - \left(\sigma_\varsigma \hat{\zeta}_t + \sigma_z \hat{H}_t\right) \tag{E.7}$$

$$\left(1-\beta\xi_w L^{-1}\right)G_t \equiv G_t - \beta\xi_w G_{t+1} = \beta\xi_w\alpha_{h,1}\widehat{\bar{w}}_{t+1} - \frac{(1-\alpha_{h,1})\,\beta\xi_w}{1-\beta\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right)$$

We now obtain a second restriction on $\hat{w}_t$ using the relation between the aggregate wage rate and the wage rates of individual households:

$$W_t = \left[(1 - \xi_w) \left(\tilde{W}_t \right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\tilde{\pi}_{w,t} W_{t-1} \right)^{\frac{1}{1-\lambda_w}} \right]^{1-\lambda_w}.$$

Dividing both sides by W_t :

$$1 = (1 - \xi_w) (w_t)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\frac{\tilde{\pi}_{w,t} W_{t-1}}{W_t} \right)^{\frac{1}{1-\lambda_w}}.$$

Note,

$$\pi_{w,t} \equiv \frac{W_t}{W_{t-1}} = \frac{\bar{w}_t z_t^+ P_t}{\bar{w}_{t-1} z_{t-1}^+ P_{t-1}} = \frac{\bar{w}_t \mu_{z^+,t} \pi_t}{\bar{w}_{t-1}},$$

so that

$$1 = (1 - \xi_w) (w_t)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\frac{\bar{w}_{t-1} \tilde{\pi}_{w,t}}{\bar{w}_t \mu_{z^+,t} \pi_t} \right)^{\frac{1}{1-\lambda_w}}.$$

Differentiate and make use of $w = 1$, $\tilde{\pi}_w = \mu_{z^+} \pi$:

$$0 = (1 - \xi_w) \frac{1}{1 - \lambda_w} \hat{w}_t + \xi_w \frac{1}{1 - \lambda_w} \left[\hat{\bar{w}}_{t-1} + \hat{\tilde{\pi}}_{w,t} - \hat{\bar{w}}_t - \hat{\mu}_{z^+,t} - \hat{\pi}_t \right],$$

or,

$$\hat{w}_t = -\frac{\xi_w}{1 - \xi_w} \left[\hat{\bar{w}}_{t-1} - \hat{\bar{w}}_t - \hat{\mu}_{z^+,t} - \Delta_{\kappa_w} \hat{\pi}_t \right].$$

Use this expression to substitute out for $\hat{w}_t$ in (E.6):

$$\frac{1 - \alpha_{h,1}}{1 - \beta \xi_w} \frac{\xi_w}{1 - \xi_w} \left[\hat{\bar{w}}_{t-1} - \hat{\bar{w}}_t - \hat{\mu}_{z^+,t} - \Delta_{\kappa_w} \hat{\pi}_t \right] = F_t + \frac{1 - \beta \xi_w \alpha_{h,1}}{1 - \beta \xi_w} \hat{\bar{w}}_t + G_t.$$

Multiply by $(1 - \beta \xi_w L^{-1})$ and use (E.7):

$$\begin{aligned} & \frac{1 - \alpha_{h,1}}{1 - \beta \xi_w} \frac{\xi_w}{1 - \xi_w} \left(1 - \beta \xi_w L^{-1} \right) \left[\hat{\bar{w}}_{t-1} - \hat{\bar{w}}_t - \hat{\mu}_{z^+,t} - \Delta_{\kappa_w} \hat{\pi}_t \right] \\ = \; & \hat{\psi}_{z^+,t} - \left(\sigma_\varsigma \hat{\zeta}_t + \sigma_z \hat{H}_t \right) + \left(1 - \beta \xi_w L^{-1} \right) \frac{1 - \beta \xi_w \alpha_{h,1}}{1 - \beta \xi_w} \hat{\bar{w}}_t \\ & + \beta \xi_w \alpha_{h,1} \hat{\bar{w}}_{t+1} - \frac{\left(1 - \alpha_{h,1} \right) \beta \xi_w}{1 - \beta \xi_w} \left(\Delta_{\kappa_w} \hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1} \right), \end{aligned}$$

or,

$$\begin{aligned} & \frac{1 - \alpha_{h,1}}{1 - \beta \xi_w} \frac{\xi_w}{1 - \xi_w} \left[\begin{array}{c} \hat{\bar{w}}_{t-1} - \beta \xi_w \hat{\bar{w}}_t - \hat{\bar{w}}_t + \beta \xi_w \hat{\bar{w}}_{t+1} - \hat{\mu}_{z^+,t} \\ + \beta \xi_w \hat{\mu}_{z^+,t+1} - \Delta_{\kappa_w} \hat{\pi}_t + \beta \xi_w \Delta_{\kappa_w} \hat{\pi}_{t+1} \end{array} \right] \\ = \; & \hat{\psi}_{z^+,t} - \left(\sigma_\varsigma \hat{\zeta}_t + \sigma_z \hat{H}_t \right) + \frac{1 - \beta \xi_w \alpha_{h,1}}{1 - \beta \xi_w} \left[\hat{\bar{w}}_t - \beta \xi_w \hat{\bar{w}}_{t+1} \right] \\ & + \beta \xi_w \alpha_{h,1} \hat{\bar{w}}_{t+1} - \frac{\left(1 - \alpha_{h,1} \right) \beta \xi_w}{1 - \beta \xi_w} \left(\Delta_{\kappa_w} \hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1} \right). \end{aligned}$$

Note that the wage does not simply enter via nominal wage inflation. To see this, note

$$\widehat{\bar{w}}_t - \widehat{\bar{w}}_{t-1} = \hat{\pi}_{w,t} - \hat{\mu}_{z^+,t} - \hat{\pi}_t,$$

where $\hat{\pi}_{w,t}$ denotes nominal wage inflation. But, it is not simply $\widehat{\bar{w}}_t - \widehat{\bar{w}}_{t-1}$ that enters in this expression. That is, if we tried to express the above expression in terms of nominal wage inflation, we would simply add another variable to it, $\hat{\pi}_{w,t}$, without subtracting any, such as the real wage, $\widehat{\bar{w}}_t$. Collecting terms:

$$\begin{aligned} 0 &= E_t[\eta_0 \widehat{\bar{w}}_{t-1} + \eta_1 \widehat{\bar{w}}_t + \eta_2 \widehat{\bar{w}}_{t+1} + \eta_3 \hat{\pi}_{t-1} + \eta_4 \hat{\pi}_t + \eta_5 \hat{\pi}_{t+1} + \eta_6 \hat{\mu}_{z^+,t} + \eta_7 \hat{\mu}_{z^+,t+1} \\ &\quad + \eta_8 \hat{\psi}_{z^+,t} + \eta_9 \hat{H}_t + \eta_{10} \hat{\zeta}_t], \end{aligned} \tag{E.8}$$

where

$$\begin{aligned} \eta_0 &= \frac{1-\alpha_{h,1}}{1-\beta\xi_w} \frac{\xi_w}{1-\xi_w},\ \eta_1 = -\eta_0 \left(1+\beta\xi_w\right) - \frac{\left(1-\beta\xi_w\alpha_{h,1}\right)}{1-\beta\xi_w}, \\ \eta_2 &= \beta\xi_w \left(\eta_0 + \frac{\left(1-\beta\xi_w\alpha_{h,1}\right)}{1-\beta\xi_w} - \alpha_{h,1}\right),\ \eta_3 = \eta_0\kappa_w, \\ \eta_4 &= -\eta_0 \left(1+\kappa_w\beta\xi_w\right) - \frac{\left(1-\alpha_{h,1}\right)\beta\xi_w}{1-\beta\xi_w}\kappa_w, \\ \eta_5 &= \eta_0\beta\xi_w + \frac{\left(1-\alpha_{h,1}\right)\beta\xi_w}{1-\beta\xi_w}, \\ \eta_6 &= -\eta_0,\ \eta_7 = \eta_5,\ \eta_8 = -1,\ \eta_9 = \sigma_z,\ \eta_{10} = \sigma_\varsigma. \end{aligned}$$

Note that (E.8) is the same for the standard model and for our model with involuntary unemployment. The difference between the two models has only to do with the construction of σ_ς and σ_z.

The wage equation can be thought of, for computational purposes, as a nonlinear equation, if we treat

$$\widehat{\bar{w}}_t = \frac{\bar{w}_t - \bar{w}}{\bar{w}},$$

and the other hatted variables in the same way.

E.3. Other Equilibrium Conditions

E.3.1. Firms

We let s_t denote the firm's marginal cost, divided by the price of the homogeneous good. The standard formula, expressing this as a function of the factor inputs, is as follows:

$$s_t = \frac{\left(\frac{r_t^k P_t}{\alpha}\right)^\alpha \left(\frac{W_t R_t^f}{1-\alpha}\right)^{1-\alpha}}{P_t z_t^{1-\alpha}}.$$

When expressed in terms of scaled variables, this reduces to:

$$s_t = \left(\frac{\bar{r}_t^k}{\alpha}\right)^{\alpha} \left(\frac{\bar{w}_t R_t^f}{1-\alpha}\right)^{1-\alpha}. \tag{E.9}$$

Productive efficiency dictates that s_t is also equal to the ratio of the real cost of labor to the marginal product of labor:

$$s_t = \frac{\left(\mu_{\Psi,t}\right)^{\alpha} \bar{w}_t R_t^f}{(1-\alpha)\left(\frac{k_{i,t}}{\mu_{z^+,t}}/H_{i,t}\right)^{\alpha}}. \tag{E.10}$$

The only real decision taken by intermediate good firms is to optimize price when it is selected to do so under the Calvo frictions. According The first order necessary conditions associated with price optimization are, after scaling:[45]

$$E_t\left[\psi_{z^+,t} y_t + \left(\frac{\tilde{\pi}_{f,t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_f}} \beta\xi_p F_{t+1}^f - F_t^f\right] = 0 \tag{E.11}$$

$$E_t\left[\lambda_f \psi_{z^+,t} y_t s_t + \beta\xi_p \left(\frac{\tilde{\pi}_{f,t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} K_{t+1}^f - K_t^f\right] = 0, \tag{E.12}$$

$$\mathring{p}_t = \left[(1-\xi_p)\left(\frac{1-\xi_p\left(\frac{\tilde{\pi}_{f,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right)^{\lambda_f} + \xi_p\left(\frac{\tilde{\pi}_{f,t}}{\pi_t}\mathring{p}_{t-1}\right)^{\frac{\lambda_f}{1-\lambda_f}}\right]^{\frac{1-\lambda_f}{\lambda_f}}, \tag{E.13}$$

$$\left[\frac{1-\xi_p\left(\frac{\tilde{\pi}_{f,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right]^{(1-\lambda_f)} = \frac{K_t^f}{F_t^f}, \tag{E.14}$$

$$\tilde{\pi}_{f,t} \equiv (\pi_{t-1})^{\kappa_d} (\pi)^{1-\kappa_d}. \tag{E.15}$$

In terms of scaled variables, the law of motion for the capital stock is as follows:

$$\bar{k}_{t+1} = \frac{1-\delta}{\mu_{z^+,t}\mu_{\Psi,t}}\bar{k}_t + \Upsilon_t\left(1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right)\right) i_t. \tag{E.16}$$

The aggregate production relation is:

$$y_t = (\mathring{p}_t)^{\frac{\lambda_f}{\lambda_f - 1}}\left[\epsilon_t\left(\frac{1}{\mu_{\Psi,t}}\frac{1}{\mu_{z^+,t}}\bar{k}_t u_t\right)^{\alpha} H_t^{1-\alpha} - \phi\right].$$

[45] When we linearize about the steady state and set $\varkappa_d = 0$, we obtain,

$$\hat{\pi}_t - \hat{\bar{\pi}}_t = \frac{\beta}{1+\kappa_d\beta}E_t\left(\hat{\pi}_{t+1} - \hat{\bar{\pi}}_{t+1}\right) + \frac{\kappa_d}{1+\kappa_d\beta}\left(\hat{\pi}_{t-1} - \hat{\bar{\pi}}_t\right) - \frac{\kappa_d\beta(1-\rho_\pi)}{1+\kappa_d\beta}\hat{\bar{\pi}}_t + \frac{1}{1+\kappa_d\beta}\frac{(1-\beta\xi_d)(1-\xi_d)}{\xi_d}\widehat{mc}_t,$$

where a hat indicates log-deviation from steady state.

Finally, the resource constraint is:

$$y_t = g_t + c_t + i_t + a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}.$$

E.3.2. Family

We now derive the equilibrium conditions associated with the household, apart from the wage condition, which was derived in a previous subsection. The Lagrangian representation of the household's problem is:

$$E_0^j \sum_{t=0}^{\infty} \beta^t \{ [\ln (C_t - bC_{t-1}) - z(h_{t,j}, \zeta_t)]$$
$$\upsilon_t \left[\begin{array}{c} W_{t,j} h_{t,j} + X_t^k \bar{K}_t + \left(R_{t-1} - \tau^R (R_{t-1} - 1)\right) B_t \\ + a_{t,j} - P_t \left(C_t + \frac{1}{\Psi_t} I_t \right) - B_{t+1} - P_t P_{k',t} \Delta_t \end{array} \right]$$
$$+\omega_t \left[\Delta_t + (1-\delta) \bar{K}_t + \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right) I_t - \bar{K}_{t+1} \right] \}$$

The first order condition with respect to C_t is:

$$\frac{1}{C_t - bC_{t-1}} - E_t \frac{b\beta}{C_{t+1} - bC_t} = \upsilon_t P_t,$$

or, after expressing this in scaled terms and multiplying by z_t^+:

$$\psi_{z^+,t} = \frac{1}{c_t - b\frac{c_{t-1}}{\mu_{z^+,t}}} - \beta b E_t \frac{1}{c_{t+1}\mu_{z^+,t+1} - bc_t}. \tag{E.17}$$

The first order condition with respect to Δ_t is, after rearranging:

$$P_t P_{k',t} = \frac{\omega_t}{\upsilon_t}. \tag{E.18}$$

The first order condition with respect to I_t is:

$$\omega_t \left[1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right) - \tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} \right] + E_t \beta \omega_{t+1} \tilde{S}'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 = \frac{P_t \upsilon_t}{\Psi_t}.$$

Making use of (E.18), multiplying by $\Psi_t z_t^+$, rearranging and using the scaled variables,

$$\begin{aligned} \psi_{z^+,t} p_{k',t} \left[1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right) - \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right)\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}} \right] & \\ +\beta \psi_{z^+,t+1} p_{k',t+1} \tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1} i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2 \mu_{z^+,t+1}\mu_{\Psi,t+1} &= \psi_{z^+,t}, \end{aligned} \tag{E.19}$$

Optimality of the choice of $\bar{K}_{t+1}$ implies the following first order condition:

$$\omega_t = \beta E_t \upsilon_{t+1} X_{t+1}^k + \beta E_t \omega_{t+1} (1-\delta) = \beta E_t \upsilon_{t+1} \left[X_{t+1}^k + P_{t+1} P_{k',t+1} (1-\delta) \right],$$

using (E.18). Using (E.18) again,

$$v_t = E_t \beta v_{t+1} \left[\frac{X_{t+1}^k + P_{t+1} P_{k',t+1} (1-\delta)}{P_t P_{k',t}} \right] = E_t \beta v_{t+1} R_{t+1}^k, \tag{E.20}$$

where R_{t+1}^k denotes the rate of return on capital:

$$R_{t+1}^k \equiv \frac{X_{t+1}^k + P_{t+1} P_{k',t+1} (1-\delta)}{P_t P_{k',t}}$$

Multiply (E.20) by $P_t z_t^+$ and express the results in scaled terms:

$$\psi_{z^+,t} = \beta E_t \psi_{z^+,t+1} \frac{R_{t+1}^k}{\pi_{t+1} \mu_{z^+,t+1}}. \tag{E.21}$$

Expressing the rate of return on capital, (3.14), in terms of scaled variables:

$$R_{t+1}^k = \frac{\pi_{t+1}}{\mu_{\Psi,t+1}} \frac{u_{t+1} \bar{r}_{t+1}^k - a(u_{t+1}) + (1-\delta) p_{k',t+1}}{p_{k',t}}. \tag{E.22}$$

The first order condition associated with capital utilization is:

$$\Psi_t r_t^k = a'(u_t),$$

or, in scaled terms,

$$\bar{r}_t^k = a'(u_t). \tag{E.23}$$

The first order condition with respect to B_{t+1} is:

$$v_t = \beta v_{t+1} R_t.$$

Multiply by $z_t^+ P_t$:

$$\psi_{z^+,t} = \beta E_t \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1} \pi_{t+1}} R_t. \tag{E.24}$$

E.4. Steady State

We describe two strategies for computing the steady state. In each case, the strategy is applied to our model with involuntary unemployment, and we indicate what changes are required for the standard model. The first steady state strategy takes all the model parameters as given and computes the endogenous variables. The second imposes values for several endogenous variables and solves for an equal number of parameters.

E.4.1. First Algorithm

Consider the equilibrium conditions associated with price setting. In steady state, these reduce to (we have used that $\tilde{\pi}^f = \pi$ using (3.4)):

$$\begin{aligned} F^f &= \frac{\psi_{z^+} y}{1-\beta\xi_p} \\ K^f &= \frac{\lambda_f \psi_{z^+} y s}{1-\beta\xi_p} \\ \mathring{p} &= 1 \\ \frac{K_t^f}{F_t^f} &= 1 \end{aligned}$$

These equations imply

$$(1) s = \frac{1}{\lambda_f}.$$

Thus,

$$\begin{aligned} (2) s &= \left(\frac{\bar{r}^k}{\alpha}\right)^{\alpha}\left(\frac{\bar{w}R^f}{1-\alpha}\right)^{1-\alpha} = \frac{1}{\lambda_f} \rightarrow \bar{w} = \frac{(1-\alpha)}{R^f}\left[\frac{1}{\lambda_f}\left(\frac{\alpha}{\bar{r}^k}\right)^{\alpha}\right]^{\frac{1}{1-\alpha}}, \\ (3) s &= \frac{(\mu_\Psi)^{\alpha}\,\bar{w}R^f}{\epsilon\,(1-\alpha)\left(\frac{\bar{k}}{\mu_{z^+}}/H\right)^{\alpha}} = \frac{1}{\lambda_f}, \rightarrow \frac{\bar{k}}{H} = \mu_{z^+}\left[\frac{\lambda_f\,(\mu_\Psi)^{\alpha}\,\bar{w}R^f}{\epsilon\,(1-\alpha)}\right]^{\frac{1}{\alpha}} \end{aligned}$$

where

$$(4) R^f = \nu^f R + 1 - \nu^f$$

The equilibrium conditions associated with wage setting is:

$$(5) \bar{w} = \lambda_w \frac{z_h}{\psi_{z^+}},$$

which is the usual wage markup equation. Also,

$$(6) h = H.$$

The consumption-compensated elasticity of labor supply is

$$(7) \sigma_z = \frac{z_{hh} h}{z_h}.$$

This reduces to σ_L in the case of the standard model, and is derived above in the case of the model with involuntary unemployment.

Also,

$$(8) \left[1 - \frac{1-\delta}{\mu_{z^+}\mu_\Psi}\right]\bar{k} = \Upsilon i,$$

and

$$
\begin{aligned}
(9)\psi_{z^+} &= \frac{1}{c}\frac{\mu_{z^+}-\beta b}{\mu_{z^+}-b}, \\
p_{k'} &= 1
\end{aligned}
$$

Consider the utilization adjustment cost function. We specify that as follows, with $\sigma_b = \bar{r}^k$:

$$
\begin{aligned}
a(u) &= 0.5\sigma_b\sigma_a u^2 + \sigma_b\left(1-\sigma_a\right)u + \sigma_b\left(\left(\sigma_a/2\right)-1\right) \\
a(1) &= \sigma_b\sigma_a + \sigma_b\left(1-\sigma_a\right) - \sigma_b
\end{aligned}
$$

Then, $a'(u) = \bar{r}^k$ implies

$$
u = 1,\ a(1) = 0.
$$

We use the latter two results in what follows. The household intertemporal Euler equation for capital implies:

$$
(10)1 = \beta\frac{R^k}{\pi\mu_{z^+}},
$$

with

$$
(11)R^k = \frac{\pi}{\mu_\Psi}\left[\bar{r}^k + 1 - \delta\right].
$$

The intertemporal Euler equation for nominal bonds:

$$
(12)1 = \frac{\beta R}{\mu_{z^+}\pi}.
$$

The production function and resource constraint:

$$
\begin{aligned}
(13)y &= \epsilon\left(\frac{1}{\mu_\Psi}\frac{1}{\mu_{z^+}}\bar{k}\right)^\alpha H^{1-\alpha} - \phi, \\
(14)y &= g + c + i
\end{aligned}
$$

The 17 variables whose steady state values are to be determined are:

$$
y, c, i, R, h, H, \bar{k}, \bar{r}^k, R^k, \bar{w}, \psi_{z^+}, s, R^f, u, \bar{p}, m, \sigma_z.
$$

Here is a strategy for solving these equations. Equation (1) produces s; R from (12); R^f from (4); R^k from (10); $\bar{r}^k$ from (11); $\bar{w}$ from (2); $\bar{k}/H$ from (3).

The remaining variables can be found using a one dimensional search. Fix a value for h. By (6), we have H; from $\bar{k}/H$ we have $\bar{k}$; from (13) we have y; from (8) we have i; from (14) and

$$
\eta_g = \frac{g}{y},
$$

we have c; from (9) we have ψ_{z+}; in the case of the model with involuntary unemployment, compute m from the steady state version of (2.12):

$$(15) h = m\eta + a^2 \varsigma \sigma_L m^{\sigma_L + 1};$$

compute σ_z using (7) or $\sigma_z = \sigma_L$ in the case of the standard model; we are now in a position to evaluate (5):

$$f(h) = \bar{w} - \lambda_w \frac{z_h}{\psi_{z+}}.$$

Adjust h until $f(h) = 0$.

In the case of our model with involuntary unemployment, we proceed as follows. Compute u from

$$(16) u = \frac{m - h}{m}.$$

Let $\bar{p}$ denote the steady state probability that the marginal household with $l = 0$ finds a job. According to (2.15):

$$(17)\, \bar{p} = \eta + \varsigma a^2 (1 + \sigma_L) m^{\sigma_L}.$$

This algorithm solves for 17 endogenous variables using 17 equations.

In the case of the standard model, m solves the steady state version of (3.23):

$$\psi_{z+} \bar{w} = \varsigma (m)^{\sigma_L},$$

and then the unemployment rate is computed using m and h.

E.4.2. A Second Algorithm

We find it convenient to shift three endogenous labor market variables to the list of exogenous variables:

$$u,\ m,\ \bar{p},\ \sigma_z.$$

Corresponding to this, we shift four parameters to the list of endogenous variables:

$$F, \varsigma, a, \eta.$$

Thus, we must solve for the following 17 variables:

$$y, c, i, R, h, H, \bar{k}, \bar{r}^k, R^k, \bar{w}, \psi_{z+}, s, R^f, F, \varsigma, a, \eta.$$

We compute the steady state as follows. As before, equation (1) produces s; R from (12); R^f from (4); R^k from (10); $\bar{r}^k$ from (11); $\bar{w}$ from (2); $\bar{k}/H$ from (3). Compute h from (16); H is found from (6); $\bar{k}$ from $\bar{k}/H$; y from (13); i from (8); c from (14) and

$$\eta_g = \frac{g}{y};$$

ψ_{z^+} from (9).

Equations (5), (7), (15), and (17) remain to be solved for F, ς, a, η. Fix values of F and a. We compute η as follows. Rewrite (15):

$$m^{\sigma_L+1} = \frac{h - m\eta}{a^2 \varsigma \sigma_L}.$$

Multiply both sides of (17) by m and substitute out for m^{σ_L+1} from the previous expression:

$$\frac{\sigma_L}{1+\sigma_L}\left(\bar{p} + \frac{\eta}{\sigma_L}\right) m = h$$

Substitute this into (16):

$$u = \frac{m-h}{m} = 1 - \frac{\sigma_L}{1+\sigma_L}\left(\bar{p} + \frac{\eta}{\sigma_L}\right) = \frac{1 + \sigma_L (1-\bar{p}) - \eta}{1+\sigma_L},$$

or,

$$\eta = 1 + \sigma_L (1 - \bar{p}) - (1 + \sigma_L) u,$$

so that we now have η (this must be non-negative). Rewriting (15), we have ς :

$$(15') \; \varsigma = \frac{h - m\eta}{a^2 m^{\sigma_L+1} \sigma_L}.$$

Next, adjust F and a so that (5) and (7) are satisfied.

E.4.3. Steady State Replacement Ratio

Here, we compute the steady state consumption of employed and non-employed households. By the analog of (2.9), the incentive constraint for the j^{th} family is:

$$c_{t,j}^{w} = bC_{t-1} + \left(c_{t,j}^{nw} - bC_{t-1}\right) e^{F + \zeta_t (1+\sigma_L) m_{t,j}^{\sigma_L}}.$$

The j^{th} family's resource constraint is:

$$h_{t,j} c_{t,j}^{w} + (1 - h_{t,j})\, c_{t,j}^{nw} = C_t.$$

Substituting out for $c_{t,j}^{w}$ from the incentive constraint, we obtain:

$$bC_{t-1} + \left(c_{t,j}^{nw} - bC_{t-1}\right) e^{F + \zeta_t (1+\sigma_L) m_{t,j}^{\sigma_L}} + \frac{(1 - h_{t,j})}{h_{t,j}} c_{t,j}^{nw} = \frac{C_t}{h_{t,j}}.$$

Rearrange,

$$c_{t,j}^{nw} = \frac{C_t + \left(e^{F + \zeta_t (1+\sigma_L) m_{t,j}^{\sigma_L}} - 1\right) h_{t,j} b C_{t-1}}{h_{t,j}\left(e^{F + \zeta_t (1+\sigma_L) m_{t,j}^{\sigma_L}} - 1\right) + 1}.$$

Scaling,

$$\tilde{c}_{t,j}^{nw} = \frac{c_t + \left(e^{F+\zeta_t(1+\sigma_L)m_{t,j}^{\sigma_L}} - 1\right) h_{t,j} \frac{b}{\mu_{z+}} c_{t-1}}{h_{t,j}\left(e^{F+\zeta_t(1+\sigma_L)m_{t,j}^{\sigma_L}} - 1\right) + 1}.$$

In steady state:

$$\tilde{c}^{nw} = c \frac{1 + \left(e^{F+\zeta(1+\sigma_L)m^{\sigma_L}} - 1\right) h \frac{b}{\mu_{z+}}}{h\left(e^{F+\zeta(1+\sigma_L)m^{\sigma_L}} - 1\right) + 1},$$

and

$$\tilde{c}^{w} = \frac{b}{\mu_{z+}} c + \left(c^{nw} - \frac{b}{\mu_{z+}} c\right) e^{F+\zeta(1+\sigma_L)m^{\sigma_L}}$$

F. Relationship of Our Work to Gali (2010)

Our paper emphasizes labor supply in its explanation of the dynamics of unemployment and the labor force. Another recent paper that adopts this perspective is Gali (2009). To better explain our model, it is useful to compare its properties with those of Gali's model. Gali demonstrates that with a modest reinterpretation of variables, the standard DSGE model already contains a theory of unemployment. In particular, one can define the unemployed as the difference between the number of people actually working and the number of people that would be working if the marginal cost of work were equated to the wage rate. This difference is positive and fluctuating in the standard DSGE model because of the presence of wage-setting frictions and monopoly power. In effect, unemployment is a symptom of social inefficiency. People inflict unemployment upon themselves in the quest for monopoly profits. By contrast, in our model unemployment reflects frictions that are necessary for people to find jobs. The existence of unemployment does not require monopoly power. This point is dramatized by the fact that we introduce our model in the CGG framework, in which wages are set in competitive labor markets. At the same time, the logic of our model does create a positive relationship between monopoly power and unemployment. In our model, the employment contraction resulting from an increase in the monopoly power of unions produces a reduction in the incentives for households to work. Households' response to the reduced incentives is to allocate less effort to search, implying higher unemployment. So, our model shares the prediction of Gali's model that unemployment should be higher in economies with more union monopoly power. However, our model has additional implications that could differentiate it from Gali's. Ours implies that in economies with more union power both the labor force and the consumption premium for employed workers over non-employed workers are reduced. Gali's model predicts that with more union monopoly power, the labor force will be larger. The exact amount by which the labor force increases depends on the strength of wealth effects on leisure.

Other important differences between our model of unemployment and Gali's is that the latter fails to satisfy characteristics (i) and (iii) above. The model assumes that the available jobs can be found without effort. Because the model does not satisfy (i), unemployment does not meet the official US definition of unemployment. In addition, the presence of perfect insurance in Gali's model implies that the employed have lower utility than the non-employed, violating (iii).

There are more differences between ours and Gali's theory unemployment. In standard DSGE models, labor supply plays little role in the dynamics of standard macro variables like consumption, output, investment, inflation and the interest rate. The reason is that the presence of wage setting frictions reduces the importance of labor supply. This is why the New Keynesian literature has been relatively unconcerned about all the old puzzles about income effects on labor and labor supply elasticities that were a central concern in the real business cycle literature. However, CTW show that these problems are back in full force if one adopts Gali's theory of unemployment. This is because labor supply corresponds to the labor force in that theory. To see how this brings back the old problems, we study the standard DSGE model's predictions for unemployment and the labor force in the wake of an expansionary monetary policy shock. Because that model predicts a rise in consumption, the model also predicts a decline in labor supply, as the income effect associated with increased consumption produces a fall in the value of work. The drop in labor supply is counterfactual, according to our VAR-based evidence. In addition, the large drop in the labor force leads to an counterfactually large drop in unemployment in the wake of an expansionary monetary policy shock.

Gali (2010), Gali, Smets and Wouters (2010) and CTW show, in different ways, that changes to the household utility function that offset wealth effects reduce the counterfactual implications of the standard model for the labor force. In effect, our paper proposes a different strategy. We preserve the additively separable utility function that is standard in monetary DSGE models, and our model nevertheless does not display the labor force problems in the standard DSGE model. This is because in our model the labor force and employment have a strong tendency to comove. In our model, the rise in employment in the wake of an expansionary monetary policy shock is accomplished by increasing people's incentives to work. The additional incentives not only encourage already active households to intensify their job search, but also to shift into the labor force. More generally, the analysis highlights the fact that modeling unemployment requires thinking carefully about the determinants of the labor force.[46]

[46] Our argument complements the argument in Krusell, Mukoyama, Rogerson, and Sahin (2009), who also

G. Aggregate Hours Worked

Given our linear approximation, and the assumptions that imply that steady state is undistorted by wage frictions, we have

$$\hat{h}_t = \hat{H}_t.$$

Although this is a well known result (see, e.g., Yun (1996)), we derive it here for completeness. Recall,

$$h_t \equiv \int_0^1 h_{j,t} dj.$$

Invert the demand for labor, (3.6), to obtain an expression in terms of $h_{j,t}$. Substitute this into the expression for h_t to obtain:

$$h_t = H_t \int_0^1 \mathring{w}_{j,t}^{\frac{\lambda_w}{1-\lambda_w}} dj, \tag{G.1}$$

where

$$\mathring{w}_{j,t} \equiv \frac{W_{j,t}}{W_t}.$$

Here, W_t denotes the aggregate wage rate, which one obtains by substituting (3.5) into (3.6):

$$W_t = \left[\int_0^1 W_{j,t}^{\frac{1}{1-\lambda_w}} dj \right]^{1-\lambda_w}.$$

Because all families are identical in steady state (see the discussion after (3.10)), $\mathring{w}_j = 1$ for all j. Totally differentiating (G.1),

$$\hat{h}_t = \hat{H}_t + \int_0^1 \widehat{\mathring{w}}_{j,t} dj.$$

Thus, to determine the percent deviation of aggregate employment from steady state, we require the integral of the percent deviations of type j wages from the aggregate wage, over all j. We now show that this integral is, to first order, equal to zero.

Express the integral in (G.1) as follows:

$$h_t = \mathring{w}_t^{\frac{\lambda_w}{1-\lambda_w}} H_t,$$

say, where

$$\mathring{w}_t \equiv \left[\int_0^1 \mathring{w}_{j,t}^{\frac{\lambda_w}{1-\lambda_w}} dj \right]^{\frac{1-\lambda_w}{\lambda_w}}. \tag{G.2}$$

stress the importance of understanding employment, unemployment and the labor force.

Pursuing logic that is standard in the Calvo price/wage setting literature we obtain:

$$W_t = \left[(1-\xi_w) \left(\tilde{W}_t \right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\tilde{\pi}_{w,t} W_{t-1} \right)^{\frac{1}{1-\lambda_w}} \right]^{1-\lambda_w} \tag{G.3}$$

$$\mathring{w}_t = \left[(1-\xi_w) w_t^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \mathring{w}_{t-1} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}}, \tag{G.4}$$

where:

$$w_t \equiv \frac{\tilde{W}_t}{W_t},\ \pi_{w,t} \equiv \frac{W_t}{W_{t-1}},$$

and $\tilde{W}_t$ denotes the wage set by the $1-\xi_w$ families that have the opportunity to reoptimize in the current period. Because all families are identical in steady state

$$w = \mathring{w} = \frac{\tilde{\pi}_w}{\pi_w} = 1, \tag{G.5}$$

where $\tilde{\pi}_{w,t}$ is defined in (3.10) and $\pi_{w,t}$ denotes wage inflation:

$$\pi_{w,t} \equiv \frac{W_t}{W_{t-1}}.$$

Dividing (G.3) by W_t and solving,

$$w_t = \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right]^{1-\lambda_w}. \tag{G.6}$$

Differentiating (G.4) and (G.6) in steady state:

$$\widehat{\mathring{w}}_t = (1-\xi_w) \hat{w}_t + \xi_w \left(\widehat{\tilde{\pi}}_{w,t} - \hat{\pi}_{w,t} + \widehat{\mathring{w}}_{t-1} \right) \tag{G.7}$$

$$\hat{w}_t = -\frac{\xi_w}{1-\xi_w} \left(\widehat{\tilde{\pi}}_{w,t} - \hat{\pi}_{w,t} \right)$$

Using the latter to substitute out for $\hat{w}_t$ in (G.7):

$$\widehat{\mathring{w}}_t = \xi_w \widehat{\mathring{w}}_{t-1}.$$

Thus, to first order the wage distortions evolve according to a stable first order difference equation, unperturbed by shocks. For this reason, we set

$$\widehat{\mathring{w}}_t = 0, \tag{G.8}$$

for all t.

Totally differentiating (G.2) and using (G.5), (G.8):

$$\int_0^1 \widehat{\mathring{w}}_{j,t} dj = 0.$$

That is, to first order, the integral of the percent deviations of individual wages from the aggregate is zero.

H. Functional Forms of Cost Functions

We adopt the following functional form for the capacity utilization cost function a :

$$a(u) = 0.5\sigma_b\sigma_a u^2 + \sigma_b (1 - \sigma_a) u + \sigma_b ((\sigma_a/2) - 1), \tag{H.1}$$

where σ_a and σ_b are the parameters of this function. The parameter, σ_b is equated to the steady state value of the rental rate of capital. $a' = \sigma_b\sigma_a u + \sigma_b (1 - \sigma_a)$.

We assume that the investment adjustment cost function takes the following form:

$$S(x) = \frac{1}{2}\left\{\exp\left[\sqrt{S''}(x - \mu_{z^+}\mu_{\Psi})\right] + \exp\left[-\sqrt{S''}(x - \mu_{z^+}\mu_{\Psi})\right] - 2\right\}, \tag{H.2}$$

where S'' is a parameter.

Table A1: Structural Parameters of Small Model Held Fixed Across Numerical Experiments

Parameter	Value	Description
β	$1.03^{-.25}$	Discount factor
g_A	1.0047	Technology growth
ξ_p	0.75	Price stickiness
λ_f	1.2	Price markup
ρ_R	0.8	Taylor rule: interest smoothing
r_π	1.5	Taylor rule: inflation
r_y	0.2	Taylor rule: output gap
η_g	0.2	Government consumption share on GDP
γ	0.001	Diffusion speed of technology into government consumption
ρ_g	0.8	AR(1) government consumption
ρ_{g_A}	0.8	AR(1) technology
ρ_ς	0.8	AR(1) disutility of labor
σ_g	0.05489[a]	Standard deviation government consumption shock
σ_a	0.00377[a]	Standard deviation technology shock
σ_R	0.00449[a]	Standard deviation monetary policy shock
σ_ς	0	Standard deviation disutiliy shock

[a] Set such that implied standard deviation of output growth equals 1% given no other shocks.

Table A2: The Impact of Imperfect Information in the Small Model

Variable	Involuntary Unemp. Model (Imperfect Information)	Full Information Model: Fixed Structural Params[b]	Full Information Model: Observational Equivalent[c]	Standard Model: Observational Equivalent[d]	Description
		Steady State Properties			
m	0.67	0.69	0.67	0.63	Labor force
h	0.63	0.68	0.63	0.63	Employment
u	0.056	0.015	0.056	n.a.	Unemployment rate
$\bar{p}$	0.95	0.99	0.95	n.a.	Max $p(e)$
$1/\sigma_z$	2.0	0.80	2.0	2.0	Family labor supply elasticity
$1/\kappa^{Okun}$	2.0	1.64	2.0	n.a.	Okun's law coefficient
c^{nw}/c^w	0.18	1.0	1.0	1.0	Replacement ratio
	0.189		n.a.	n.a.	Price (% of C) of info.[a]
		Structural Parameters[e]			
a	0.53	0.53	0.74	n.a.	Slope, $p(e)$
η	0.86	0.86	0.86	n.a.	Intercept, $p(e)$
ς	4.64	4.64	2.45	1.67	Slope, labor disutility
F	1.39	1.39	1.83	n.a.	Intercept, labor disutility
σ_L	13.31	13.31	13.31	0.5	Power, labor disutility
		Welfare Cost of Business Cycles			
		Technology shock only			
λ	0.520684131141	0.566191290230	0.520684131141	0.520684131142	% of consumption
		Government consumption shock only			
λ	0.112215458271	0.125326644511	0.112215458271	0.112215458271	% of consumption
		Monetary policy shock only			
λ	0.071331553871	0.100111000086	0.071331553871	0.071331553871	% of consumption

[a] Percent increase in consumption in steady state of involuntary unemployment model that makes steady state utility in that model equal to steady state utility of model with full information.

[b] Full information model with same structural parameters as involuntary unemployment model.

[c] Full information model with parameter values in Table A1, plus parameter values in bottom panel of this table, chosen so that full information model steady state properties in the top panel (except c^{nw}/c^w) coincide with those in involuntary unemployment model.

[d] Standard model with parameter values in Table A1, plus parameter values for ς and σ_L so that h and $1/\sigma_z$ coincide with those of the involuntary unemployment model.

[e] Model structural parameter values are those listed in Table A1 plus the ones indicated in the bottom panel of this table.

Table A3: Benchmark Parameter Values and Steady State Properties of the Small Model for the Sensitivity Analysis of the Costs of Business Cycles

Variable Name	Value	Description
β	$1.03^{-.25}$	Discount factor
g_A	1.0047	Technology growth
ξ_p	0.75	Price stickiness
λ_f	1.2	Price markup
ρ_R	0.8	Taylor rule: interest smoothing
r_π	1.5	Taylor rule: inflation
r_y	0.2	Taylor rule: output gap
$1/\sigma_z$	1	Family labor supply elasticity
η_g	0.2	Government consumption share on GDP
u	0.055	Unemployment rate
m	2/3	Labor force participation
$\overline{p}$	0.97	max p(e)
γ	0.001	Diffusion speed of technology into gov. spending
F	0.31	Intercept, labor disutility
σ_L	4	Power, labor disutility
ς	0.99	Slope, labor disutility
a	0.36	Slope, $p(e)$
η	0.85	Intercept, $p(e)$
ρ_g	0.8	AR(1) government consumption
ρ_{g_A}	0.8	AR(1) technology
ρ_ς	0.8	AR(1) disutility of labor
σ_g	0.06045[a]	Standard deviation government consumption shock
σ_a	0.00373[a]	Standard deviation technology shock
σ_R	0.00449[a]	Standard deviation monetary policy shock
σ_ς	0	Standard deviation disutiliy shock

[a] Set such that implied standard deviation of output growth equals 1% given no other shocks.

Table A4a: Welfare Costs of Business Cycles in the Small Model: Consumption Equivalents in %

Model	Benchmark	ξ_p=0	$r_\pi = 100$	$\eta_g = 0$	$\gamma = 1$
			Technology Shocks		
Standard Model	0.54145973949766	0.48843255026769	0.48856383934627	0.03602282953172	0.04299383343424
Involuntary Unemp.	0.54145973949775	0.48843255026769	0.48856383934645	0.03602282953172	0.04299383343424
Full Information	0.54145973949761	0.48843255026775	0.48856383934459	0.03602282953172	0.04299383343424
			Government Spending Shocks		
Standard Model	0.14695322840805	0.13822951084114	0.13825099674533	n.a.	0.14695322840805
Involuntary Unemp.	0.14695322840805	0.13822951084114	0.13825099674533	n.a.	0.14695322840805
Full Information	0.14695322840803	0.13822951084114	0.13825099674531	n.a.	0.14695322840803
			Monetary Policy Shocks		
Standard Model	0.09736866565077	n.a.	0.00023970621749	0.08157775324686	0.09736866565077
Involuntary Unemp.	0.09736866565077	n.a.	0.00023970621749	0.08157775324686	0.09736866565077
Full Information	0.09736866565075	n.a.	0.00023970621749	0.08157775324686	0.09736866565075

Table A4b: Welfare Costs of Business Cycles in the Small Model:
Consumption Equivalents in %

Model	Benchmark	$\sigma_L = 20, \sigma_z = 1$	$\sigma_L = 1.5, \sigma_z = 1$	$\sigma_L = 4, \sigma_z = 3$	$\sigma_L = 4, \sigma_z = 0.1$
			Technology Shocks		
Standard Model	0.54145973949766	0.54145973949766	0.54145973949766	0.62045916723039	0.47453492071094
Involuntary Unemp.	0.54145973949775	0.54145973949786	0.54145973949753	0.62045916723059	0.47453492071103
Full Information	0.54145973949761	0.54145973949788	0.54145973949699	0.62045916723036	0.47453492071116
			Government Spending Shocks		
Standard Model	0.14695322840805	0.14695322840805	0.14695322840805	0.18461277975589	0.12684875171187
Involuntary Unemp.	0.14695322840805	0.14695322840807	0.14695322840805	0.18461277975589	0.12684875171187
Full Information	0.14695322840803	0.14695322840807	0.14695322840805	0.18461277975589	0.12684875171187
			Monetary Policy Shocks		
Standard Model	0.09736866565077	0.09736866565077	0.09736866565077	0.16925151157873	0.05852588699248
Involuntary Unemp.	0.09736866565077	0.09736866565088	0.09736866565075	0.16925151157873	0.05852588699248
Full Information	0.09736866565075	0.09736866565086	0.09736866565077	0.16925151157873	0.05852588699248