

# The Cost of Uncertainty about the Timing of Social Security Reform\*

Frank N. Caliendo<sup>†</sup>   Aspen Gorry<sup>‡</sup>   Sita Slavov<sup>§</sup>

This Version: September 16, 2015. First Version: November 5, 2013.

## Abstract

We develop a model to study optimal decision making in the face of uncertainty about the timing and structure of a future event. The model is used to study optimal decision making and welfare when individuals face uncertainty about *when* and *how* Social Security will be reformed. When individuals save optimally for retirement, the welfare cost of uncertainty about the timing and structure of reform is just a few basis points of total lifetime consumption. In contrast, the cost of reform uncertainty can be greater than 1% of total lifetime consumption for individuals who do not save.

Key words: Policy Uncertainty, Social Security Reform, Retirement Saving, Welfare.

JEL Codes: E21, E60, H30, H55, C61.

---

\*We thank Shantanu Bagchi, Jaroslav Borovička, Peter Bossaerts, Thorsten Drautzburg, Eric Fisher, Jim Feigenbaum, Carlos Garriga, Nick Guo, Tim Kehoe, Sagiri Kitao, David Laibson, Lee Lockwood, Ezra Oberfield, Jorge Alonso Ortiz, Will Peterman, Kerk Phillips, Andrew Samwick, Jennifer Ward-Batts, Jackie Zhou, and seminar audiences at ITAM, UConn, Midwest Macro at Minnesota, Utah State University, BYU-USU Macro Workshop, University of Nevada Reno, CUNY Hunter College, WEAI conference, QSPS workshop, Second Annual Macroeconomics and Business CYCLE Conference in Santa Barbara, Stanford Institute for Theoretical Economics, Econometric Society World Congress in Montreal, and Montana State University for helpful comments and suggestions.

<sup>†</sup>Utah State University. Email: frank.caliendo@usu.edu.

<sup>‡</sup>Utah State University. Email: aspen.gorry@usu.edu.

<sup>§</sup>George Mason University. Email: sslavov@gmu.edu.

## 1. Introduction

Saving optimally for retirement is difficult because individuals face uncertainty along many dimensions. Longevity, medical expenses, future health status, earnings, and returns on savings vary for each individual. Optimal retirement saving requires understanding the distributions of these uncertainties and then forming optimal consumption/saving plans.<sup>1</sup> Beyond these recognized challenges, the long-term insolvency of the Social Security system creates an additional level of uncertainty for individuals because they do not know their future benefit levels or tax rates. This paper studies how policy uncertainty about the timing and structure of Social Security reform influences individual decision making and welfare.

Understanding how to measure policy uncertainty and how it affects economic decisions is a priority in macroeconomics (Sargent (2005), Fernández-Villaverde et al. (2013), Baker, Bloom and Davis (2013), Baker et al. (2014)).<sup>2</sup> In particular, uncertainty is generated as individuals do not know the path of future policy until policy makers act. We develop a dynamic setting to study optimal decision making when individuals do not know when or how policy reform will occur.<sup>3</sup> A feature of our setting is that it is flexible enough to handle any distribution over the resolution of timing uncertainty.

Although the set of potential Social Security reforms is known—either benefits must fall or taxes must rise—which combination of these measures policy makers will ultimately pursue and when reform will happen are not. We solve and simulate optimal consumption/saving decisions for a 25 year old individual starting with no assets and facing mortality risk over a finite lifespan. We abstract from factor price risk to focus on policy uncertainty and we consider settings where reform at an unknown date is a proportional benefit cut, a proportional tax increase, or an unknown combination of the two. In each case the reform is parameterized to make Social Security solvent over an infinite horizon.

After characterizing optimal decision making, we calculate the cost of policy uncertainty about the timing and structure of Social Security reform in the U.S. when individuals follow the optimal consumption/saving path. The cost is defined as the fraction of lifetime consumption that a young individual facing reform uncertainty would be willing to give up to live in a world with no reform uncertainty where she is endowed with her expected lifetime income over all possible realizations of reform. Our welfare

---

<sup>1</sup>This task can be challenging not just because of limits on information, but also because of limits on self control (Laibson (1997)) and limits on computational ability (Thaler (1994)).

<sup>2</sup>Some readers may prefer to reserve the word *uncertainty* for situations in which probabilities of random events are unknown and *risk* for situations in which probabilities are known. In this paper we deal only with known probabilities, and we use the words risk and uncertainty interchangeably.

<sup>3</sup>Stokey (2014), Bi, Leeper and Leith (2012), Davig, Leeper and Walker (2010), and Davig and Foerster (2014) use similar methods to study uncertainty about fiscal policy reform.

measure captures the full value of insuring against timing uncertainty. It also nets out the cost of reform itself allowing us to isolate the cost of policy uncertainty. We first consider timing uncertainty alone where either a benefit cut or a tax increase balances the Social Security budget. Next, we consider uncertainty about both the timing and structure of reform where the Social Security budget is balanced with an unknown combination of benefit cuts and tax increases.

For those who save optimally, the welfare cost is small regardless of whether reform is a tax increase, a benefit cut, or an uncertain combination of the two. For realistically parameterized reforms, if individuals know the distribution of reform dates and structures then they can appropriately save to make the costs of uncertainty small. Optimal saving behavior provides a benchmark for how low welfare costs can be for an individual facing reform uncertainty. Tax increases and benefit cuts have different effects for low-income and high-income individuals. If individuals know that the Social Security budget will be balanced with a benefit cut, then uncertainty about the timing of reform is regressive. Alternatively, when the budget will be balanced using a tax increase, then high-income individuals face larger welfare losses because they face the same percentage tax increase but rely less on the progressive benefit. Additionally, uncertainty about both the timing and structure of reform imposes larger welfare costs on low-income individuals across all parameterizations of the model.

We also consider the welfare cost to individuals who do not save at all. These individuals consume their disposable income in each period and rely exclusively on Social Security benefits during retirement. This exercise provides a natural benchmark of how high welfare costs can be when individuals do not use saving to hedge the reform uncertainty they face. This benchmark is also relevant as Hurst (2006) and others document that as many as 20-30% of Americans do not save. We find that non-savers experience welfare costs that can exceed 1% of their total lifetime consumption. For those who imperfectly hedge the reform uncertainty that they face, possibly due to under-saving or not fully understanding the nature of the uncertainty, their saving behavior would fall between optimal and non-saving behavior. We conclude that the welfare cost of policy uncertainty about Social Security reform can be quite large, relative to a world in which a non-saver consumes with certainty her expected income over all possible realizations of reform.

Because these baseline exercises correspond to a 25 year old individual starting with no assets, we explore how initial age and assets influence the cost of living with uncertainty about the timing and structure of Social Security reform. In every case, higher levels of wealth reduce the cost of uncertainty as the possible benefit cut or tax increase is less important to the individual. Moreover, timing uncertainty is more costly for older individuals in both the case of a certain benefit cut and when there is uncertainty

about the structure of reform as they have less time to prepare for a future reduction in benefits. For instance, timing uncertainty about a benefit cut costs a 65 year old with no current assets nearly 5% of remaining lifetime consumption if an optimal plan is followed thereafter and over 7.5% for a non-saver. In contrast, uncertainty is less costly for older workers in cases of a tax increase as the increase only affects a shorter portion of their life and they have a higher probability of being unaffected by the policy change.

We also extend our analysis to consider a grandfathering policy that exempts individuals from reform once they reach age 55. While individuals are better off under such a policy since they have higher expected lifetime income, grandfathering does not dramatically reduce the costs of timing uncertainty. In fact, the costs of uncertainty about the timing of reform are higher in most cases with grandfathering than without. This is because grandfathering actually makes the distribution of timing risk that young individuals face *more* severe, eliminating intermediate cases since either all or none of Social Security benefits are cut.

Our results are related to a small literature that seeks to quantify the costs of Social Security reform uncertainty. In perhaps the most closely related work to ours, Gomes, Kotlikoff and Viceira (2007) study a model with uncertainty about whether a benefit cut will occur at a given future date. Their baseline individual would be willing to give up 0.12% of annual consumption in exchange for learning about the cut to Social Security benefits at age 35 instead of age 65. While this result has a similar flavor to ours, individuals in their model never actually face timing uncertainty because the date at which information is released is known in advance and welfare is measured as a comparison between early and late resolution of uncertainty.<sup>4</sup> Benítez-Silva et al. (2007) also compute the welfare loss from having to live with uncertainty about the future level of Social Security benefits, but there is no timing uncertainty in their model either. Bütler (1999) studies the welfare cost of uncertainty about the timing of public pension reform in Switzerland but does not consider the distribution of welfare costs across income groups.

Taking an alternative approach, Luttmer and Samwick (2012) use survey data to elicit the degree of Social Security reform uncertainty that individuals perceive and how costly such uncertainty is to them. They find that individuals would be willing to tolerate an additional 4-6% cut in benefits in exchange for certainty about their level.

---

<sup>4</sup>Evans, Kotlikoff and Phillips (2012) consider a model in which future transfers to the old are uncertain because transfers are based on the stochastic wages of the young. Likewise, van der Wiel (2008) considers the effect of uncertainty about future Social Security benefits on private savings, but similar to Gomes, Kotlikoff and Viceira (2007), the individual knows in advance that the government will announce the new level of benefits at the date of retirement, so there is no timing uncertainty.

For the theory, we combine and generalize existing tools from the two-stage optimal control literature that deal independently with either timing uncertainty or with structural uncertainty but not both at once. Examples of two-stage control problems with a stochastic regime switch date (timing uncertainty) can be found in studies on resource extraction (Dasgupta and Heal (1974)), operations research (Kamien and Schwartz (1971)), and environmental catastrophe (Clarke and Reed (1994)). And examples of problems with uncertainty about the characteristics of the new regime (structural uncertainty) appeared in the resource extraction literature (Hoel (1978)) and later in the technology adoption literature (Hugonnier, Pelgrin and Pommeret (2006), Pommeret and Schubert (2009), Abel and Eberly (2012)).

Finally, our paper is related to a literature on long-run risk. For example, Epstein, Farhi and Strzalecki (2014) compute the “timing premium” which is the amount households would be willing to pay to immediately resolve uncertainty about future consumption. But there is no timing uncertainty in their setting. Individuals always know in advance whether they are living in the early-resolution world or the late-resolution world. Their experiment is to calculate how much households in the second world would pay to live in the first.<sup>5</sup> The preference for early resolution in their setting is driven by a recursive utility structure. We are interested in situations in which decision makers not only face uncertainty about the outcome of a future event, but the timing of that event is also uncertain. In contrast to Epstein, Farhi and Strzalecki (2014), we avoid recursive utility in favor of standard CRRA preferences. Of course, in our model the cost of policy uncertainty would be much larger with recursive preferences because in that case individuals would pay for early information even if they could not use that information to reoptimize.

## 2. Theory: Timing Uncertainty and Structural Uncertainty

We begin by solving a dynamic problem that features timing uncertainty only. Then we solve a problem with both timing and structural uncertainty.<sup>6</sup> Optimal planning in the face of timing and structural uncertainty requires that decision makers compute contingent plans for every possible realization of the date and structure of reform. Decision makers must then embed these contingent plans into an ex ante problem that assigns a continuation value to the state variable based on the probability of each

---

<sup>5</sup>Blundell and Stoker (1999) and Eeckhoudt, Gollier and Treich (2005) study the connection between optimal consumption and the timing of income risk. They compare the case in which income risk gets resolved early to the case in which income risk gets resolved late. Rather than modeling timing uncertainty, this literature focuses on understanding how early versus late resolution of uncertainty affects decision making. In each case, the timing of the resolution of uncertainty is known in advance. Likewise, Wright, Bloom and Barrero (2014) extend these ideas to firm investment behavior.

<sup>6</sup>Our analysis builds on the two-stage control literature in which the switch date is deterministic and may be a choice variable or exogenous (Kemp and Long (1977), Tomiyama (1985), Amit (1986), Tahvonen and Withagen (1996), Makris (2001), Boucekkine, Saglam and Vallee (2004), Dogan, Van and Saglam (2011), Saglam (2011), Boucekkine, Pommeret and Prieur (2012), Boucekkine, Pommeret and Prieur (2013a), and Boucekkine, Pommeret and Prieur (2013b)).

contingency.

Time is continuous and indexed by  $t$ . Time starts at  $t = 0$  and never ends. The planning interval of the decision maker, which begins at  $t = 0$  and ends at  $t = T$ , is comprised of two possible regimes or stages. Each stage has a unique performance index and/or state equation. The regime switch date  $t_1$  is a continuous random variable, with probability density  $\phi(t_1)$  and support on  $[0, \infty]$ . This probability density is known at date zero and no additional information about the timing of reform is revealed over time, except that the shock has not yet hit. We allow for the possibility that the shock  $t_1$  hits after  $T$  and therefore the regime switch is never experienced by the decision maker. The control variable  $u(t)$  is unconstrained and the state variable  $x(t)$  is constrained only at  $t = 0$  and  $t = T$ ,  $x(0) = x_0$  and  $x(T) = x_T$ .

A feature of our setting is that it is flexible enough to handle non-stationary distributions over the resolution of timing uncertainty, which contrasts with the standard stochastic dynamic programming setting in which uncertainty is represented as a stationary process that depends only on the state.<sup>7</sup>

The stochastic regime switching control problem can be solved recursively in two steps.<sup>8</sup>

### 2.1. Step 1: Post-switch ( $t = t_1$ ) subproblem

The first step of the recursive procedure is to solve a deterministic control problem from the vantage point of the switch date  $t_1$ , taking as given the timing of the switch  $t_1$  and the quantity of the state variable at the switch date  $x(t_1)$ . The program  $(u_2^*(t|t_1, x(t_1)), x_2^*(t|t_1, x(t_1)))_{t \in [t_1, T]}$  solves a fixed endpoint Pontryagin subproblem:

$$\max_{u(t)_{t \in [t_1, T]}} : J_2 = \int_{t_1}^T f_2(t, u(t), x(t)) dt, \quad (1)$$

subject to

$$\frac{dx(t)}{dt} = g_2(t, u(t), x(t)|t_1), \text{ for } t \in [t_1, T], \quad (2)$$

$$t_1 \text{ given, } x(t_1) \text{ given, } x(T) = x_T. \quad (3)$$

The payoff function  $f_2$  and the state function  $g_2$  are continuously differentiable in their arguments. Given the Hamiltonian function,

$$\mathcal{H}_2 = f_2(t, u(t), x(t)) + \lambda_2(t)g_2(t, u(t), x(t)|t_1), \quad (4)$$

---

<sup>7</sup>While it may be that one can characterize uncertainty about the timing of a regime switch as a state dependent process, it is convenient to specify the distribution of switch dates as a function of time so that we can quickly consider a wide variety of distributions without having to reconsider the appropriate state space.

<sup>8</sup>See Appendix A for a full derivation of the solution.

the necessary conditions that hold on the path  $(u_2^*(t|t_1, x(t_1)), x_2^*(t|t_1, x(t_1)))_{t \in [t_1, T]}$  include  $\partial \mathcal{H}_2 / \partial u(t) = 0$  and  $d\lambda_2(t)/dt = -\partial \mathcal{H}_2 / \partial x(t)$ . For convenience, change the time dummy  $t$  to  $z$ , and change the switch point  $t_1$  to  $t$  and write the solution  $(u_2^*(z|t, x(t)), x_2^*(z|t, x(t)))_{z \in [t, T]}$ . Thus we have the optimal control and state paths for all points in time  $z$  greater than switch point  $t$ .

## 2.2. Step 2: Pre-switch ( $t = 0$ ) subproblem

Working backwards, the next step is to solve the control problem from the perspective of time 0, using the solution from the previous step to construct a continuation function that links the two problems together. The program  $(u_1^*(t), x_1^*(t))_{t \in [0, T]}$  solves a fixed endpoint Pontryagin subproblem with continuation function  $\mathcal{S}(t, x(t))$ :

$$\max_{u(t)_{t \in [0, T]}} : J_1 = \int_0^T \left\{ \left[ \int_t^\infty \phi(t_1) dt_1 \right] f_1(t, u(t), x(t)) + \phi(t) \mathcal{S}(t, x(t)) \right\} dt, \quad (5)$$

subject to

$$\mathcal{S}(t, x(t)) = \int_t^T f_2(z, u_2^*(z|t, x(t)), x_2^*(z|t, x(t))) dz, \quad (6)$$

$$\frac{dx(t)}{dt} = g_1(t, u(t), x(t)), \text{ for } t \in [0, T], \quad (7)$$

$$x(0) = x_0, \quad x(T) = x_T. \quad (8)$$

The payoff function  $f_1$  and the state function  $g_1$  are continuously differentiable in their arguments. The functions  $f_1$  and  $f_2$  could change form at the regime switch date, although in our applications  $f_1$  and  $f_2$  are the same. The state functions  $g_1$  and  $g_2$  change form at the switch date in our applications. Given the Hamiltonian function,

$$\mathcal{H}_1 = \left[ \int_t^\infty \phi(t_1) dt_1 \right] f_1(t, u(t), x(t)) + \phi(t) \mathcal{S}(t, x(t)) + \lambda_1(t) g_1(t, u(t), x(t)), \quad (9)$$

the necessary conditions that must hold on the path  $(u_1^*(t), x_1^*(t))_{t \in [0, T]}$  include  $\partial \mathcal{H}_1 / \partial u(t) = 0$  and  $d\lambda_1(t)/dt = -\partial \mathcal{H}_1 / \partial x(t)$ .

To summarize, the optimal control and state paths are  $(u_2^*(t|t_1, x(t_1)), x_2^*(t|t_1, x(t_1)))_{t \in [t_1, T]}$  for any  $t$  *after* the realization of the random regime switch, conditional on the switch date  $t_1$  and conditional on the stock of the state variable at that switch date  $x(t_1)$ . Similarly,  $(u_1^*(t), x_1^*(t))_{t \in [0, T]}$  are the optimal control and state paths for any  $t$  *before* the realization of the random switch. Hence, the path that is actually followed, conditional on switch date  $t_1$ , is  $(u_1^*(t), x_1^*(t))_{t \in [0, t_1]}$  and  $(u_2^*(t|t_1, x_1^*(t_1)), x_2^*(t|t_1, x_1^*(t_1)))_{t \in [t_1, T]}$ .

Mangasarian (1966) shows that if  $g_1$  and  $g_2$  are linear in  $u(t)$  and  $x(t)$  and the integrands of  $J_1$  and

$J_2$  are concave in  $u(t)$  and  $x(t)$ , then the necessary conditions are sufficient. Checking the concavity of the integrand of  $J_2$  is standard. But checking the concavity of the integrand of  $J_1$  is more involved. This is because the integrand of  $J_1$  depends on the optimal post-switch path. Thus, one must first derive the post-switch solution  $(u_2^*(z|t, x(t)), x_2^*(z|t, x(t)))_{z \in [t, T]}$ , which depends on  $x(t)$ , and then insert this solution into  $\mathcal{S}$  before checking the concavity of the integrand of  $J_1$ .

### 2.3. Adding Structural Uncertainty

In addition to stochastic *timing* of the regime switch, we can easily allow for the possibility that the *structure* of the new regime itself (the functional form of the post-switch state equation) is uncertain. Adding in this second layer of uncertainty is relatively easy and requires just a few adjustments to the previous method.<sup>9</sup>

To add structural uncertainty, we assume that the uncertainty about the functional form of  $g_2$  is summarized by the random variable  $\alpha$ , with density  $\theta(\alpha)$  and support on  $[0, 1]$ , where  $\theta(\alpha)$  is continuously differentiable and realizations of  $\alpha$  and  $t_1$  are uncorrelated. The necessary conditions are again derived recursively with only slight modifications: in Step 1 use the notation  $g_2(t, u(t), x(t)|t_1, \alpha)$  and  $(u_2^*(t|t_1, x(t_1), \alpha), x_2^*(t|t_1, x(t_1), \alpha))_{t \in [t_1, T]}$  to emphasize dependence of the solution on the realization of  $\alpha$ , and in Step 2 write the continuation function  $\mathcal{S}(t, x(t), \alpha)$  and replace the last term in the integrand of  $J_1$  with  $\int_0^1 \theta(\alpha) \phi(t) \mathcal{S}(t, x(t), \alpha) d\alpha$ .

## 3. Application 1: Uncertainty about the Timing of Reform

The Social Security program in the U.S. faces severe long run solvency problems. The 2014 Social Security Trustees Report projects that the Old-Age and Survivors Insurance (OASI) trust fund will run out of money in the year 2034. This means that over the coming decades either promised retirement benefits must be cut or the payroll taxes used to fund them must be increased to keep the program solvent. Gokhale (2013) estimates that an immediate tax increase of 3.1% of taxable wages, or an immediate benefit cut of 21%, will keep the OASI program solvent for the infinite horizon.<sup>10</sup>

While there is very little uncertainty about the need for reform, there is a great deal of uncertainty surrounding its timing and structure. For instance, Sargent (2005) states: “*We do not know today...how*

---

<sup>9</sup>Models with uncertainty about the structure of the new regime appeared in the early resource extraction literature (Hoel (1978)) and then again in the more modern literature on technology adoption when future returns to technology are stochastic (Hugonnier, Pelgrin and Pommeret (2006), Pommeret and Schubert (2009), Abel and Eberly (2012)).

<sup>10</sup>If the disability component of the program is also included, then the required tax increase (assuming no behavioral response) is 4% and the required benefit cut is 23.9% as estimated in the 2013 Trustees Report.



*subsequent political deliberations from shifting majority coalitions will render U.S. fiscal policy coherent.”*<sup>11</sup>

And even though the feasibility/optimalty of various Social Security reforms have been widely studied in macroeconomic models (e.g., McGrattan and Prescott (2014) and Kitao (2014) and the references therein), this literature treats the timing and structure of reform as part of the household information set. We focus on the primary question of measuring welfare costs to a single individual in an environment where the timing and structure of reform are unknown.<sup>12</sup>

We begin with an application of uncertainty only about the timing of reform because timing uncertainty is more novel and less is known about its welfare consequences. In this scenario, the individual has full information about the structure of reform, and we consider the possibility of either a benefit cut or a tax increase to render Social Security solvent occurring at an unknown date.

### 3.1. Notation

Age is continuous and is indexed by  $t$ . Households are born at  $t = 0$  and pass away no later than  $t = T$ . The probability of surviving to age  $t$  is  $\Psi(t)$ . Retirement and benefit collection occur exogenously at  $t = t_R$  and labor is supplied inelastically.<sup>13</sup> A given household collects wages at rate  $w(t)$  during the working period.

The government’s *current* policy is summarized by a tax rate on wage earnings and benefit annuity  $(\tau_1, b_1)$ . There is public knowledge that the current policy is unsustainable. Therefore households know that reform is coming, but they don’t know when. The reform date  $t_1$  is a random variable with probability density  $\phi(t_1)$  and support on  $[0, \infty]$ . The post-reform policy is  $(\tau_2(t_1), b_2(t_1))$ . For now, we assume households have full information about the nature of the reform, they just don’t know when it will kick in.

---

<sup>11</sup>This uncertainty arises not only from the reluctance of elected officials to propose unpopular reforms, but also from ongoing disagreements over which reform option is most desirable. In particular, Democrats have tended to favor tax increases, while Republicans have tended to favor benefit cuts. For more detail, see recent legislation introduced by members of Congress and summarized by the Office of the Chief Actuary, at [www.ssa.gov](http://www.ssa.gov).

<sup>12</sup>While Bütler (1999) studies a particular example of the implications of uncertainty about both the timing and structure of social security reform in Switzerland, our methodology allows us to handle any assumptions about the distributions of both structural and timing uncertainty. This flexibility allows us to study timing and structural uncertainty in a general way and to explore the distributional effects of reform in the US.

<sup>13</sup>People predominantly retire from the labor force at the early and normal eligibility ages (see Diamond and Gruber (1999) among others). Fixing the retirement and collection dates allows us to parameterize the model to empirically reasonable values for these choices while abstracting from natural and institutional complications that shape the incentives behind these choices.

Let  $y_1(t)$  be disposable income before the reform and let  $y_2(t|t_1)$  be disposable income after the reform,

$$y_1(t)_{t \in [0, t_1]} = \begin{cases} (1 - \tau_1)w(t), & \text{for } t \in [0, t_R], \\ b_1, & \text{for } t \in [t_R, T], \end{cases} \quad (10)$$

$$y_2(t|t_1)_{t \in [t_1, T]} = \begin{cases} (1 - \tau_2(t_1))w(t), & \text{for } t \in [0, t_R], \\ b_2(t_1), & \text{for } t \in [t_R, T]. \end{cases} \quad (11)$$

In this experiment the individual is still subject to a benefit cut even after the retirement date. We will consider grandfathering in an extension later in the paper.

Consumption is  $c(t)$  and savings is  $k(t)$ , which earns interest at rate  $r$ . The only constraints on savings are that initial assets are zero and the individual cannot plan to leave behind debt at the maximum lifespan, hence  $k(0) = 0$  and  $k(T) = 0$ .

### 3.2. Household Behavior

Period utility is CRRA,  $c(t)^{1-\sigma}/(1-\sigma)$ , with relative risk aversion  $\sigma$ , and utils are discounted exponentially at the rate of time preference  $\rho$ . In Appendix B we provide a step-by-step derivation of the solution to a dynamic stochastic utility maximization problem for which the date of reform is a random variable. Here, we simply report the solution.

The optimal pre-reform solution  $(c_1^*(t), k_1^*(t))_{t \in [0, T]}$  solves the following system of differential equations and boundary conditions:

$$\begin{aligned} \frac{dc(t)}{dt} &= \left( \frac{c(t)^{\sigma+1}}{\Psi(t)} \left[ \frac{k(t) + \int_t^T e^{-r(v-t)} y_2(v|t) dv}{\int_t^T e^{-r(v-t) + (r-\rho)v/\sigma} \Psi(v)^{1/\sigma} dv} \right]^{-\sigma} e^{(\rho-r)t} - c(t) \right) \times \left[ \frac{\sigma}{\phi(t)} \int_t^\infty \phi(t_1) dt_1 \right]^{-1} \\ &+ \left[ \frac{d\Psi(t)}{dt} \frac{1}{\Psi(t)} + r - \rho \right] \frac{c(t)}{\sigma}, \end{aligned} \quad (12)$$

$$\frac{dk(t)}{dt} = rk(t) + y_1(t) - c(t), \quad (13)$$

$$k(0) = 0, \quad k(T) = 0. \quad (14)$$

The individual follows this path up to the random reform date  $t_1$ . The optimal post-reform (after the shock has hit) consumption path is

$$c_2^*(t|t_1, k_1^*(t_1)) = \frac{k_1^*(t_1) + \int_{t_1}^T e^{-r(v-t_1)} y_2(v|t_1) dv}{\int_{t_1}^T e^{-r(v-t_1) + (r-\rho)v/\sigma} \Psi(v)^{1/\sigma} dv} e^{(r-\rho)t/\sigma} \Psi(t)^{1/\sigma}, \quad \text{for } t \in [t_1, T]. \quad (15)$$

Note that consumption after reform depends on the timing of reform and on the stock of assets at the time of reform.

### 3.3. Welfare

The welfare cost of reform uncertainty is defined as the fraction of lifetime consumption that a young individual facing reform uncertainty would be willing to give up to live in a separate world with no reform uncertainty and endowed with his expected wealth over all possible realizations of reform dates. Our welfare measure calculates the value of insuring against timing uncertainty because the no-risk world guarantees the individual a deterministic consumption path that is based on expected wealth over different realizations of the reform date. Our measure also nets out the cost of reform itself, allowing us to isolate the cost of this specific example of policy uncertainty, as we already know from Kitao (2014) and others that the welfare cost of reform itself is large.<sup>14</sup>

To compute the welfare cost of policy uncertainty, consider the case where the individual faces no risk (NR) about future taxes and benefits and is endowed at  $t = 0$  with the present discounted value of her expected future income. She solves

$$\max_{c(t)_{t \in [0, T]}} : \int_0^T e^{-\rho t} \Psi(t) \frac{c(t)^{1-\sigma}}{1-\sigma} dt, \quad (16)$$

subject to

$$\frac{dk(t)}{dt} = rk(t) - c(t), \quad (17)$$

$$k(0) = \int_0^T \phi(t_1) Y(t_1) dt_1 + \left[ \int_T^\infty \phi(t_1) dt_1 \right] \int_0^T e^{-rv} y_1(v) dv, \quad k(T) = 0, \quad (18)$$

where

$$Y(t_1) \equiv \int_0^{t_1} e^{-rv} y_1(v) dv + \int_{t_1}^T e^{-rv} y_2(v|t_1) dv. \quad (19)$$

---

<sup>14</sup>Bütler (1999) studies the welfare cost of uncertainty about the timing of public pension reform in Switzerland. However, she compares the welfare of individuals who must live with timing uncertainty to individuals who live in a separate world with no timing uncertainty and with reform that is guaranteed to happen at the date that is mathematically expected in the first world. While such a comparison captures important differences in behavior, it does not pin down the cost of uncertainty. Instead, it confounds changes in wealth with the effects of uncertainty. In fact, it is theoretically possible to come to the mistaken conclusion that uncertainty actually is a good thing in that environment (because it could increase an individual's expected lifetime wealth). For instance, if individuals in the first world are uncertain about when a benefit cut will strike and the mathematical expectation is that it will strike just before retirement, then individuals would want maximum variance around this expectation in order to create the possibility of collecting full benefits for at least some portion of the retirement period.

The solution is

$$c^{NR}(t) = \frac{\int_0^T \phi(t_1)Y(t_1)dt_1 + [\int_T^\infty \phi(t_1)dt_1] \int_0^T e^{-rv}y_1(v)dv}{\int_0^T e^{-rv+(r-\rho)v/\sigma}\Psi(v)^{1/\sigma}dv} e^{(r-\rho)t/\sigma}\Psi(t)^{1/\sigma}, \text{ for } t \in [0, T]. \quad (20)$$

For those who save optimally, the welfare cost of living with reform uncertainty  $\Delta^S$  is the compensation that equates utility from the no-risk consumption stream (left hand side) to expected utility under uncertainty (right hand side):

$$\begin{aligned} & \int_0^T e^{-\rho t}\Psi(t) \frac{[c^{NR}(t)(1 - \Delta^S)]^{1-\sigma}}{1 - \sigma} dt \\ = & \int_0^T \phi(t_1) \left( \int_0^{t_1} e^{-\rho t}\Psi(t) \frac{c_1^*(t)^{1-\sigma}}{1 - \sigma} dt + \int_{t_1}^T e^{-\rho t}\Psi(t) \frac{c_2^*(t|t_1, k_1^*(t_1))^{1-\sigma}}{1 - \sigma} dt \right) dt_1 \\ & + \left[ \int_T^\infty \phi(t_1)dt_1 \right] \int_0^T e^{-\rho t}\Psi(t) \frac{c_1^*(t)^{1-\sigma}}{1 - \sigma} dt. \end{aligned} \quad (21)$$

Notice that  $\Delta^S$  is the fraction of consumption that the individual would be willing to give up to live in a world with no timing uncertainty. By endowing the individual with expected wealth over all possible reform dates, we are calculating how much the individual would be willing to pay to fully insure against the effects timing uncertainty.

Our welfare metric  $\Delta^S$  is similar to what Epstein, Farhi and Strzalecki (2014) call the “timing premium,” though in the models that they consider people are willing to pay for early resolution of uncertainty because of the way utility is specified (Epstein-Zin), even if the early information cannot be used to reoptimize. This contrasts with our CRRA setting in which early resolution leads to welfare gains in part because it allows for better optimization and smaller distortions to consumption/saving decisions. This distinction arises because Epstein, Farhi and Strzalecki (2014) consider uncertainty over *consumption* streams whereas we consider uncertainty over *income* streams. If we were to use Epstein-Zin utility then our timing premium would be larger as it would capture both the distortions to consumption and saving caused by late resolution of uncertainty and also the innate desire to know one’s consumption outcomes in advance. Epstein, Farhi and Strzalecki (2014) remain skeptical, however, that people would be willing to pay very much to know their consumption outcomes in advance if there is nothing that can be done to change those outcomes.

We compare our welfare measure for a saver to the cost when the individual does not save. In this case, the individual sets consumption equal to disposable income in each period. Results for the non-saver document how large the welfare cost of uncertainty about the timing of reform can be when the individual

does not respond at all to uncertainty. This contrasts with the baseline case where the individual forms optimal consumption/saving plans to hedge the risk from reform uncertainty.

For a non-saver, the welfare cost of living with uncertainty about the timing of reform  $\Delta^N$  solves the same equation as before, but updated to include the no-saving constraint:

$$\begin{aligned} & \int_0^T e^{-\rho t} \Psi(t) \frac{[c^{NR}(t)(1 - \Delta^N)]^{1-\sigma}}{1 - \sigma} dt \\ = & \int_0^T \phi(t_1) \left( \int_0^{t_1} e^{-\rho t} \Psi(t) \frac{y_1(t)^{1-\sigma}}{1 - \sigma} dt + \int_{t_1}^T e^{-\rho t} \Psi(t) \frac{y_2(t|t_1)^{1-\sigma}}{1 - \sigma} dt \right) dt_1 \\ & + \left[ \int_T^\infty \phi(t_1) dt_1 \right] \int_0^T e^{-\rho t} \Psi(t) \frac{y_1(t)^{1-\sigma}}{1 - \sigma} dt \end{aligned} \quad (21')$$

where

$$c^{NR}(t) = \left[ \int_t^\infty \phi(t_1) dt_1 \right] y_1(t) + \int_0^t \phi(t_1) y_2(t|t_1) dt_1. \quad (20')$$

Note that we endow the individual in the no-risk world with a consumption path that equals his expected disposable income at each age, which is a weighted average of  $y_1(t)$  and  $y_2(t|t_1)$ .

For both savers and non-savers, our welfare measure follows in the tradition of calculating willingness-to-pay to avoid uncertainty by comparing expected utility to utility from expected wealth. However, the welfare cost of timing uncertainty could also be calculated by comparing expected utility to ex ante expected utility in a world in which the date of Social Security reform is announced at time zero. In this case, the individual knows that she will follow the optimal deterministic consumption path conditional on a particular reform date, but ex ante she doesn't know which deterministic path she will follow. This alternative measure also captures the value of knowing the date of reform in advance, but it does not allow the individual to insure her wealth across different realizations of the timing of reform. As a result, this alternative measure is guaranteed by Jensen's inequality to produce a smaller welfare cost from timing uncertainty than our measure.

Finally, with this alternative measure, non-savers would by definition experience zero welfare loss from policy uncertainty. This is because non-savers follow the same consumption path no matter when information about timing uncertainty is announced. In contrast, in our welfare calculations non-savers prefer a world without uncertainty because their wealth is fully insured. While our results below emphasize that non-savers have larger welfare losses than savers, this conclusion is reversed for this alternative welfare measure.

### 3.4. Parameterization

The model is parameterized to capture individual income levels and survival probabilities over the life cycle. The parameters to be chosen are the maximum lifespan  $T$ , the survival function  $\Psi(t)$ , the exogenous retirement date  $t_R$ , the real return on assets  $r$ , the individual discount rate  $\rho$ , the coefficient of relative risk aversion  $\sigma$ , the age-earnings distribution  $w(t)$ , the probability density over reform dates  $\phi(t_1)$ , and policy parameters capturing tax rates and benefit levels before and after reform  $\{\tau_1, \tau_2(t_1), b_1, b_2(t_1)\}$ .

Our survival data come from the Social Security Administration’s cohort mortality tables. These tables contain the mortality assumptions underlying the intermediate projections in the 2013 Trustees Report. The mortality table for each cohort provides the number of survivors at each age  $\{1, 2, \dots, 119\}$ , starting with a cohort of 10,000 newborns. However, we truncate the mortality data at age 100, assuming that everyone who survives to age 99 dies within the next year. In the baseline results, we assume individuals enter the labor market at age 25, giving them a 75-year potential lifespan within the model. In our baseline parameterization, we use the mortality profile for males born in 1990, who are assumed to enter the labor market in 2015. For this cohort, we construct the survival probabilities at all subsequent ages conditional on surviving to age 25.

We normalize time so that the maximum age in the model is  $T = 1$ . Thus  $t = 0$  in the model corresponds to age 25, and  $t = 1$  corresponds to age 100. Because the survival data are discrete (providing the probability of surviving to each integer age), we fit a continuous survival function that has the following form:

$$\Psi(t) = 1 - t^x. \tag{22}$$

After transforming the survival data to correspond to model time, with dates on  $[0, 1]$ ,  $x = 3.28$  provides the best fit to the data.

The fixed retirement age is assumed to occur at age 65, which corresponds to  $t_R = \frac{40}{75}$  in the model.<sup>15</sup> We assume a risk-free real interest rate of 2.9% per year, which is the long-run real interest rate assumed by the Social Security Trustees. In our model, this implies a value of  $r = 75 * 0.029 = 2.175$ . Estimates of the individual discount rate  $\rho$  vary substantially in the literature, and values of  $\rho < r$  are necessary to

---

<sup>15</sup>While the Social Security normal retirement age is 66 for cohorts born between 1943 and 1954, and will gradually rise to 67 for cohorts born in 1960 and later, we use 65 as the exogenous retirement age for a few reasons. First, income data from Gourinchas and Parker (2002) is only available until age 65. Second, many individuals stop working and claim an actuarially reduced Social Security benefit before the normal retirement age. Finally, this assumption can make our results easier to compare with previous research, as many prior studies specify a retirement age of 65. This assumption will also be important when setting replacement rates for individuals of different incomes. We use replacement rates corresponding to retirement at age 65, rather than normal retirement age.

generate a hump shaped consumption profile in the model. In the baseline model we set  $\rho = 0$ , although we consider other values in robustness exercises. In the baseline calibration we also set  $\sigma = 3$ , with other values considered for robustness.

We import the individual income profile from Gourinchas and Parker (2002) with age normalized onto model time  $[0, 1]$  and the maximum income normalized to one. The continuous-time wage function is approximated by fitting a fifth-order polynomial to the discrete-time wage data:

$$w(t) = 0.697 + 1.49t - 3.41t^2 + 19.08t^3 - 59.78t^4 + 52.70t^5. \quad (23)$$

Figure 1 shows the graphs of the wage profile and the survival probabilities.

There is not much evidence about the distribution of possible reform dates, as this depends on subjective expectations about the political process.<sup>16</sup> Although the Social Security trust fund runs out in 2034, uncertainty about reform may extend beyond that date. For example, policy makers may adopt a temporary fix as 2034 approaches, postponing major reform even further into the future. Or, perhaps policy makers will work together to address reform well in advance of 2034.<sup>17</sup>

Given the lack of evidence, we consider a few different distributions of  $\phi(t_1)$  to understand the implications of the distribution of timing uncertainty on individual welfare. To begin, we assume that reform is a Weibull random variable,

$$\phi(t_1) = \frac{\mu}{\gamma} \left( \frac{t_1}{\gamma} \right)^{\mu-1} e^{-(t_1/\gamma)^\mu}, \text{ for } t_1 \in [0, \infty]. \quad (24)$$

We consider two special cases, the exponential density and the Rayleigh density. First, we assume  $\mu = 1$  to generate a constant hazard rate of reform:

$$\phi(t_1) = \frac{e^{-t_1/\gamma}}{\gamma}. \quad (25)$$

Because it seems unlikely that the individual will totally escape reform, we calibrate this function by

---

<sup>16</sup>We depart from a literature that assumes individuals don't fully *understand* the existing rules of Social Security, while the political process is stationary and the rules are knowable (Liebman and Luttmer (2014)). Instead, in our setting the future of Social Security is unknowable.

<sup>17</sup>The Health and Retirement Study (HRS), an ongoing panel survey of older Americans, regularly asks respondents to rate the chances of a cut in Social Security benefits within the next 10 years. In the 2012 wave of the survey, the mean subjective probability of a benefit cut within the next 10 years is around 67%; however, there is much variance around this value. The Survey of Economic Expectations also elicits information on household expectations about future Social Security benefits (Dominitz, Manski and Heinz (2003), Manski (2004)). While this survey does document substantial uncertainty, it does not specifically measure uncertainty about the timing of reform.

assuming  $\int_1^\infty (e^{-t_1/\gamma})/\gamma dt_1 = 1\%$ , which implies  $\gamma = -1/\ln 0.01$ .

Alternatively, it is plausible that political pressure for reform will mount as the trust fund runs out of money by 2034. For this reason we also consider a second calibration where the likelihood of reform rises as the trust fund exhaustion date approaches. To capture this, we compute  $\phi'(t_1) = 0$  and set  $t_1 = (2034 - 2015)/75$ , which implies

$$\gamma = \frac{19}{75} \left( \frac{\mu}{\mu - 1} \right)^{\frac{1}{\mu}}. \quad (26)$$

The larger the value of  $\mu$ , the greater the mass around 2034. Our computational procedure struggles with values of  $\mu$  larger than 2, so we set  $\mu = 2$  which then implies  $\gamma = 19/75 \times \sqrt{2}$ . Figure 2 shows the graphs of these two calibrations of  $\phi(t_1)$ . We will report welfare calculations for both calibrations.

The *current* Social Security policy  $(\tau_1, b_1)$  in the model is parameterized to match the current policy in the U.S. Consistent with our modeling in earlier sections, we focus only on retirement insurance (the Old Age and Survivors, or OASI, program) and ignore disability insurance. The OASI payroll tax rate (combined employer and employee shares) is  $\tau_1 = 10.6\%$ . The employee pays the full tax in our model because labor is inelastic. Benefits  $b_1$  are chosen to match observed replacement rates for various income groups.

Social Security benefits are based on an individual's Average Indexed Monthly Earnings (AIME), calculated as average monthly earnings, indexed for economy-wide wage growth, over the highest 35 years of the individual's career. A progressive benefit formula is applied to AIME to arrive at an individual's Primary Insurance Amount (PIA), the monthly benefit payable if benefits are claimed at normal retirement age. Claiming before normal retirement age—for example, at age 65, as we assume in our model—results in an actuarial reduction to benefits. The progressive benefit formula implies that the replacement rate—the ratio of monthly benefits to AIME—falls with AIME.

The Social Security Trustees Report publishes replacement rates for several stylized workers, each earning a fixed multiple of the economy-wide average wage throughout their career. According to the 2013 Trustees Report, the very low income group, which has career-average earnings equal to 25% of the economy-wide average wage, receives a replacement rate of 67.5% of AIME if benefits are claimed at age 65. The low-income group, which has career-average earnings equal to 45% of the economy-wide average wage, has a replacement rate of 49.0% of AIME. The medium income group, which has career-average earnings equal to the economy-wide average wage, receives a replacement rate of 36.4% of AIME. The high income group, which has career-average earnings equal to 1.6 times the average wage, receives a



replacement rate of 30.1% of AIME. Finally, workers who earn the maximum taxable amount in each year of their career receive a replacement rate of 24.0% of AIME. These replacement rates apply to the year 2055, when the 1990 cohort turns 65. To compute pre-reform benefits,  $b_1$ , we apply these replacement rates to the AIME for the normalized life-cycle income profile in our model, which is 0.92927.<sup>18</sup>

For the policy experiments considered in this paper we assume that the reform will balance the Social Security budget over the infinite horizon. While the Social Security Trustees reports provide detailed estimates of long-run funding shortfalls for the combined Old-Age, Survivors, and Disability Insurance (OASDI) program, obtaining estimates for the OASI program alone is more challenging. Gokhale (2013) estimates that the infinite horizon OASI funding shortfall beginning in 2012 amounts to \$15.9 trillion, representing 3.1% of the present value of taxable wages over that period (which Gokhale estimates as \$505.7 trillion). Thus, with no behavioral response, a tax increase of 3.1 percentage points would be needed to close the funding shortfall. Gokhale also estimates that a 21% benefit reduction would be required to eliminate the infinite horizon shortfall.<sup>19</sup>

To determine how the required benefit cut and tax increase change if the government postpones reform, we combine these estimates with the 75-year horizon estimates of taxable wages and benefits provided in the 2012 Trustees Report. The Trustees Report estimates that the present value of taxable wages over 2012-2086 is \$341.5 trillion. Combined with Gokhale’s infinite horizon estimate, this suggests that the 2012 present value of taxable wages over 2087 through the infinite horizon is \$505.7 trillion - \$341.5 trillion = \$164.2 trillion. If no changes are made before that point, the infinite horizon shortfall remains the same in 2012 present value, as funds would need to be borrowed to cover any shortfall over 2012-2086. Thus, the required tax increase in 2087 would be \$15.9 trillion / \$164.2 trillion = 9.7% of wages. This calculation suggests that the required tax increase rises by 6.6 percentage points over 75 years (which coincides with the time elapsed between model time 0 and model time 1). Thus, a linear approximation of the post-reform tax rate is  $\tau_2(t_1) = \tau_1 + 3.1\% + 6.6\% \times t_1$ . Note that we ignore the slight increase in the required tax hike between 2012 and 2015.

For benefit cuts, the 2012 Trustees Report estimates that the present value of OASI costs are \$48.8

---

<sup>18</sup>In computing AIME for our life-cycle model, we disregard wage indexation because the Gourinchas and Parker (2002) real income profiles are already adjusted not only for price inflation, but also for cohort and time effects.

<sup>19</sup>Gokhale’s estimate of the present value of taxable wages differs somewhat from the estimate provided in the 2012 Trustees Report, which is \$530.2 trillion. Of course, the required tax and benefit adjustments could be much different in a macroeconomic model that includes household behavioral responses and factor price responses to demographic and fiscal shocks. For instance, in Kitao (2014) the required tax increase is 6% and the required benefit cut is 33%. Our welfare costs go up when we feed bigger policy shocks such as these into our model. While the Trustees Reports take into account behavioral responses in their estimates of the tax increase required to close the combined OASDI shortfall, they do not provide similar estimates for OASI alone.

trillion over the following 75 years. The OASI unfunded liability as a share of these costs is 15.19%. The Trustees report does not provide an estimate of the benefit cut required to close the OASI 75-year shortfall. However, the report notes that the combined OASDI program has 75-year unfunded liabilities of \$8.6 trillion, representing 15.23% of the present value of combined OASDI costs over the period and requiring a 16.2% benefit cut to eliminate.<sup>20</sup> That is, the required benefit cut is 1.064 (16.2% / 15.23%) times the unfunded liability as a share of costs. If we assume that this ratio holds over any horizon, Gokhale’s estimate of a 21% benefit cut implies that the infinite horizon unfunded liability amounts to 19.74% of costs, and that infinite horizon costs are 15.9 trillion / 19.74% = \$80.5 trillion. Thus, costs over 2087 through the infinite horizon are \$80.5 trillion - \$48.8 trillion = \$31.7 trillion. Unfunded liabilities are \$15.9 trillion / \$31.7 trillion = 50.1% of this amount, suggesting that the required benefit cut in 2087 is  $1.064 \times 50.1\% = 53\%$ . Since the required benefit cut rises by 32 percentage points over 75 years, a linear approximation of the new benefit level is  $b_2(t_1) = b_1 \times (1 - 21\% - 32\% \times t_1)$ .

By parameterizing tax increases and benefit cuts in a realistic way that is consistent with available evidence and actuarial projections, we are potentially understating the costs of uncertainty. We assume that Social Security will continue to exist and that reforms will be modest, whereas in reality a large portion of young households are not sure if Social Security will exist at all when they retire (Dominitz, Manski and Heinz (2003)). For instance, Luttmer and Samwick (2012) elicit willingness to pay to remove reform uncertainty by asking individuals to answer survey questions in which there is a non-trivial chance that Social Security will be eliminated completely. Individuals in our model are not worried about such radical reform risk and therefore our welfare costs may understate the costs associated with individuals’ perceived uncertainty about reform.

As an initial pass, we consider two possible reforms:

- **Full benefit cut.** An across-the-board reduction in benefits to  $b_2(t_1)$  for all current and future retirees (no exemption for current retirees) and no change in taxes.
- **Full tax increase.** An across-the-board increase in the OASI tax rate to  $\tau_2(t_1)$  for all taxpayers regardless of age and no change in benefits.

We consider each reform separately and assume that individuals know which reform will occur, but they are uncertain about the timing. The results from these scenarios clarify the intuition of how timing

---

<sup>20</sup>It is not stated in the Trustees report whether this estimate assumes any behavioral response. We suspect the required benefit cut may also differ from the unfunded liability as a share of costs because not all costs are benefit payments, and because cutting benefits may affect revenue via the taxation of benefits.

uncertainty influences individual consumption and saving decisions. We will later consider the case where individuals are also uncertain about the structure of reform, not knowing if it will be a benefit cut, a tax increase, or some combination of the two.

### 3.5. Results

The optimal consumption/saving rules have been solved analytically, up to an unknown constant  $c(0)$ . To simulate results we guess and iterate on  $c(0)$  until the boundary constraints are satisfied.

The results for the benefit cut and tax increase scenarios are plotted to gain intuition about the effects of uncertainty about the timing of reform. Figure 3 shows consumption profiles over the life cycle for the full benefit cut experiment for an individual with average earnings who faces a constant hazard rate of reform. Four profiles are plotted: the optimal consumption path conditional on still being in the pre-reform regime  $c_1^*$ ; a pair of optimal post-reform consumption paths,  $c_2^*$ , conditional on reform at  $t_1 = 0.5$  and  $t_1 = 0.8$  as examples; and the consumption profile from a world with no Social Security risk where the individual is endowed upfront with expected lifetime income,  $c^{NR}$ . The path the individual actually follows is  $c_1^*$  up to the stochastic date of reform and then consumption drops to  $c_2^*$ . Although there are many  $c_2^*$  paths, we only plot two possibilities for illustrative purposes. This figure shows how uncertainty about the timing of benefit reform causes non-trivial distortions to consumption-saving decisions. If the reform shock is realized early, then consumption falls below the level without Social Security risk, whereas a later reform shock leaves consumption above the no-risk path.

Figure 4 plots the same information but for the case of a tax increase, again for an individual with average earnings who faces a constant hazard rate of reform. We show two hypothetical reform dates,  $t_1 = 0.2$  and  $t_1 = 0.5$  as examples. Reform dates after retirement are not plotted because even though reform may strike after retirement, there is no distortion to consumption since individuals no longer pay Social Security taxes when they are not working. Similar to the case of a benefit cut, consumption always drops at the moment reform strikes.

The drop in consumption in either scenario is the result of rational, forward-looking behavior in the face of uncertainty. Reconsider Figure 3 and suppose the individual is standing just to the left of  $t = 0.8$ . From this perspective, the individual knows the shock may happen at any time over the interval  $[0.8, \infty]$  and therefore bases consumption on that expectation, which is rational ex ante. But if the shock hits at the next moment (i.e., at  $t = 0.8$ ) then ex post the individual turns out to be a little poorer than anticipated the moment before and hence consumption must be revised down. Individuals are always surprised the moment reform occurs.

Table 1 compares the welfare cost of timing uncertainty between individuals who save optimally and those who do not save, for five different income groups. Comparisons are shown for the case of a benefit cut as well as the case of a tax increase. Recall that in this exercise we assume that the individual only faces timing uncertainty and therefore knows the structure of reform in each case.

Among those who save optimally, the welfare cost of timing uncertainty is just a few basis points of total lifetime consumption. The exact value within this range depends on the income group and on whether reform is a benefit cut or a tax increase. In all cases, the magnitude of the welfare cost is relatively small.

We also find important distributional effects as we look beyond the average individual. Focusing on those who save optimally, uncertainty about the timing of a benefit cut is more harmful to low income individuals than to high income individuals. This is due to the progressivity of benefits. For example, Table 1 shows that, for the case of constant hazard rate of reform (Panel A), the very low income group will experience a welfare loss that is more than 4 times larger than what is experienced by the highest income group that maximizes their Social Security contributions in each year of work. This asymmetry occurs because Social Security benefits are a larger share of total retirement income for the poor than for the rich, and uncertainty over something important is going to be more costly. This result continues to hold in Panel B when we replace the exponential distribution with one that peaks at 2034.

But this distributional effect reverses its sign for the case of uncertainty about the timing of a tax increase. Essentially, now the progressivity argument works in the opposite way. Even though all income groups pay the same tax rate, uncertainty about the tax rate is more costly for the rich because Social Security benefits are smaller for them (relative to their wage) and hence they face uncertainty about a larger portion of their wealth than do the poor. This result holds across both assumptions about the distribution of reform shocks.

The most interesting results are found in the comparison of savers to non-savers. In the case of a benefit cut, non-savers experience welfare costs that can exceed 1% of total lifetime consumption. This is roughly 1 or 2 *orders of magnitude* larger than the costs to those who save optimally. Not knowing when a benefit cut will occur is very costly when benefits are the sole source of retirement income, and simply having access to capital markets allows optimal savers to hedge away most of the timing risk. The regressivity of timing uncertainty is reversed for non-savers. This reversal arises because replacement rates are lower for high income groups. But comparisons across income groups is probably less interesting in this case because income may be an important determinant of saving behavior.

Timing uncertainty about a tax increase, however, imposes costs on non-savers that are in the same

ballpark as the costs imposed on savers. In both cases, the welfare cost of timing uncertainty is small. This result holds for both assumptions about the distribution of reform shocks.

#### 4. Application 2: Timing and Structural Uncertainty

Now we consider the case where both the timing and structure of Social Security reform are uncertain. First we introduce notation and welfare, and then we compare simulated welfare costs from this application to the previous application with only timing uncertainty.

##### 4.1. Notation and Welfare

Let  $\tilde{\tau}_2(t_1)$  be the new tax rate that would be sufficient to balance the budget without any reduction in benefits, and likewise let  $\tilde{b}_2(t_1)$  be the new benefit level that would balance the budget without a tax increase. Of course,  $\tilde{\tau}_2(t_1) > \tau_1$  and  $\tilde{b}_2(t_1) < b_1$ . But the new policy that the government actually chooses is an uncertain, linear combination of these extremes. We will express the new tax policy as a function of a continuous random variable  $\alpha$  with density  $\theta(\alpha)$  and support on  $[0, 1]$ :

$$\tau_2(t_1, \alpha) = \tau_1 + \alpha(\tilde{\tau}_2(t_1) - \tau_1), \quad (27)$$

$$b_2(t_1, \alpha) = b_1 - (1 - \alpha)(b_1 - \tilde{b}_2(t_1)), \quad (28)$$

and

$$y_2(t|t_1, \alpha) = \begin{cases} (1 - \tau_2(t_1, \alpha))w(t), & \text{for } t \in [0, t_R], \\ b_2(t_1, \alpha), & \text{for } t \in [t_R, T]. \end{cases} \quad (29)$$

Suppressing the derivation, the pre-reform Euler equation is

$$\begin{aligned} \frac{dc(t)}{dt} &= \left( \frac{c(t)^{\sigma+1}}{\Psi(t)} \int_0^1 \theta(\alpha) \left[ \frac{k(t) + \int_t^T e^{-r(v-t)} y_2(v|t, \alpha) dv}{\int_t^T e^{-r(v-t) + (r-\rho)v/\sigma} \Psi(v)^{1/\sigma} dv} \right]^{-\sigma} e^{(\rho-r)t} d\alpha - c(t) \right) \\ &\times \left[ \frac{\sigma}{\phi(t)} \int_t^\infty \phi(t_1) dt_1 \right]^{-1} \\ &+ \left[ \frac{d\Psi(t)}{dt} \frac{1}{\Psi(t)} + r - \rho \right] \frac{c(t)}{\sigma}. \end{aligned} \quad (30)$$

Using this Euler equation, together with the law of motion and boundary conditions for the savings account, we can compute the stage-one solution  $(c_1^*(t), k_1^*(t))_{t \in [0, T]}$ . The post-reform consumption path

is

$$c_2^*(t|t_1, k_1^*(t_1), \alpha) = \frac{k_1^*(t_1) + \int_{t_1}^T e^{-r(v-t_1)} y_2(v|t_1, \alpha) dv}{\int_{t_1}^T e^{-r(v-t_1)+(r-\rho)v/\sigma} \Psi(v)^{1/\sigma} dv} e^{(r-\rho)t/\sigma} \Psi(t)^{1/\sigma}, \text{ for } t \in [t_1, T]. \quad (31)$$

Finally, for welfare comparisons, the no risk benchmark is

$$c^{NR}(t) = \frac{\int_0^1 \int_0^T \theta(\alpha) \phi(t_1) Y(t_1|\alpha) dt_1 d\alpha + \left[ \int_T^\infty \phi(t_1) dt_1 \right] \int_0^T e^{-rv} y_1(v) dv}{\int_0^T e^{-rv+(r-\rho)v/\sigma} \Psi(v)^{1/\sigma} dv} e^{(r-\rho)t/\sigma} \Psi(t)^{1/\sigma}, \quad t \in [0, T], \quad (32)$$

where  $Y(t_1|\alpha) \equiv \int_0^{t_1} e^{-rv} y_1(v) dv + \int_{t_1}^T e^{-rv} y_2(v|t_1, \alpha) dv$ , and the welfare cost to savers of living with reform uncertainty  $\Delta^S$  is the compensation that equates utility from expected income over all possible reform dates and structures (left hand side) to expected utility (right hand side):

$$\begin{aligned} & \int_0^T e^{-\rho t} \Psi(t) \frac{[c^{NR}(t)(1 - \Delta^S)]^{1-\sigma}}{1 - \sigma} dt \\ = & \int_0^1 \int_0^T \theta(\alpha) \phi(t_1) \left( \int_0^{t_1} e^{-\rho t} \Psi(t) \frac{c_1^*(t)^{1-\sigma}}{1 - \sigma} dt + \int_{t_1}^T e^{-\rho t} \Psi(t) \frac{c_2^*(t|t_1, k_1^*(t_1), \alpha)^{1-\sigma}}{1 - \sigma} dt \right) dt_1 d\alpha \\ & + \left[ \int_T^\infty \phi(t_1) dt_1 \right] \int_0^T e^{-\rho t} \Psi(t) \frac{c_1^*(t)^{1-\sigma}}{1 - \sigma} dt. \end{aligned} \quad (33)$$

As in Application 1 above, we wish to compare the welfare cost experienced by optimal savers to the cost experienced by non-savers. For non-savers, the welfare cost of living with uncertainty about the timing and structure of reform  $\Delta^N$  solves the same equation as before, but updated to include the no-saving constraint

$$\begin{aligned} & \int_0^T e^{-\rho t} \Psi(t) \frac{[c^{NR}(t)(1 - \Delta^N)]^{1-\sigma}}{1 - \sigma} dt \\ = & \int_0^1 \int_0^T \theta(\alpha) \phi(t_1) \left( \int_0^{t_1} e^{-\rho t} \Psi(t) \frac{y_1(t)^{1-\sigma}}{1 - \sigma} dt + \int_{t_1}^T e^{-\rho t} \Psi(t) \frac{y_2(t|t_1, \alpha)^{1-\sigma}}{1 - \sigma} dt \right) dt_1 d\alpha \\ & + \left[ \int_T^\infty \phi(t_1) dt_1 \right] \int_0^T e^{-\rho t} \Psi(t) \frac{y_1(t)^{1-\sigma}}{1 - \sigma} dt \end{aligned} \quad (33')$$

where

$$c^{NR}(t) = \left[ \int_t^\infty \phi(t_1) dt_1 \right] y_1(t) + \int_0^1 \int_0^t \theta(\alpha) \phi(t_1) y_2(t|t_1, \alpha) dt_1 d\alpha. \quad (32')$$

Note that as before the individual in the no-risk world is endowed with a consumption path that equals expected disposable income at each age, which is a weighted average of  $y_1(t)$  and  $y_2(t|t_1, \alpha)$ .

In the absence of reliable data on expectations about the structure of future reform, which is ultimately

a political decision that will reflect the preferences of future policy makers, we assume  $\theta(\alpha)$  is the uniform density. In an extension later in the paper we will explore other assumptions.

## 4.2. Results

Table 2 augments the information in Table 1 to include the welfare costs of double uncertainty. For those who save optimally, the welfare cost of double uncertainty is similar to the welfare cost under just timing uncertainty only. And, as in the case with only timing uncertainty, the welfare cost experienced by non-savers can be more than 1% of total lifetime consumption. Not knowing when or how reform will occur is very costly when benefits are the sole source of retirement income. Having access to capital markets allows optimal savers to hedge away much of the costly uncertainty that they face.

Figure 5 shows the effects of double uncertainty on consumption allocations over the life cycle of an average earner facing a constant hazard rate of reform. As with previous figures, we plot the optimal consumption path given that reform has not yet happened,  $c_1^*$ , the consumption path corresponding to a hypothetical world without reform risk,  $c^{NR}$ , and many post-reform consumption profiles  $c_2^*$  conditional on reform striking at various dates. We show the following reform dates  $t_1 \in \{0.1, 0.2, \dots, 0.8, 0.9\}$ , and we show just three particular realizations of the structure of reform  $\alpha \in \{0, 0.5, 1\}$  for each of the reform dates. Although our welfare calculations take into account that the timing ( $t_1$ ) and structure ( $\alpha$ ) of reform are continuous random variables, we plot just a few realizations to highlight the types of consumption paths that individuals may experience.

By showing some of the contingent consumption plans formulated by the individual, Figure 5 illustrates the breadth of the distortions to consumption caused by the presence of double uncertainty about Social Security reform. For instance, the distortion to consumption in the middle of the retirement period is especially severe. The positive consumption spikes result from the cases in which the individual draws a full-tax-reform shock (with no benefit cut). This is a positive shock to expected wealth during retirement since the individual learns that there is no risk of a benefit cut. Consumption drops in cases where the individual draws a full-benefit-reform shock, eliminating the hope of escaping benefit reform.

Whereas uncertainty about the timing of reform always triggers downward corrections in consumption, with double uncertainty the consumption correction can be positive. This is because a tax increase is good news for individuals near retirement or already in retirement. Now that they know their benefits are safe, they scale up their consumption spending in response to the positive income shock.

## 5. Extensions

In this section we show how our results change when we alter certain assumptions. Unless we say otherwise, we assume baseline parameterizations for the distributions of timing uncertainty (exponential distribution) and structural uncertainty (uniform distribution).

### 5.1. Age and Wealth Heterogeneity

The parameterizations that we have considered so far are based on the expected utility of an individual who enters the labor force in 2015 at age 25 with no assets. Of course, there are many other individuals at different ages and levels of accumulated savings who face uncertainty about the future of the Social Security system. This section evaluates how the welfare effects of reform uncertainty depend on the age and assets of the individual.

We will use the same notation and methods as above, and we will interpret time zero as any arbitrary older age. Let  $t_0 \in (0, 1)$  be the fraction of the total lifespan that the individual has already lived. We perform the following normalization of parameters in order to normalize the individual's current age  $t_0$  to zero, so that time runs on  $[0, 1]$  regardless of where in the life cycle the individual is currently standing. First, we parameterize the retirement age  $t_R$  as the fraction of the remaining total lifespan spent working. Second, we estimate new survival functions  $\Psi(t)$  on  $[0, 1]$  that use survival probabilities conditional on surviving to the current age  $t_0$ . We fit the survival function  $\Psi(t) = 1 - t^x$  to the appropriate cohort in the underlying mortality data, e.g., 45 year olds in 2015 were born in 1970. Third, we modify the wage distribution by replacing all  $t$  on the right-hand-side of  $w(t)$  with  $\hat{t}(t) \equiv t_0 + t(1 - t_0)$ . Fourth, we replace all  $t_1$  on the right-hand-side of  $\phi(t_1)$ ,  $\tau_2(t_1)$ , and  $b_2(t_1)$  with  $t_1(1 - t_0)$ . In the case of  $\phi(t_1)$ , we also renormalize the height of the p.d.f. to ensure unit mass under the curve. We set  $\gamma = -1/\ln 0.01$  as before. Fifth, we set  $r = (1 - t_0) \times 2.175$ .

We consider the welfare costs for individuals beginning at age 45, 55, and 65 (model ages  $t_0 = 0.27, t_0 = 0.4, t_0 = 0.53$ ). In each case, we consider five levels of initial assets. For each age, the baseline asset level is the assets that the individual in our initial exercise for a 25 year old worker would have accumulated if reform has not hit by the older age,  $k_1^*(t_0)$ . Beyond this baseline level, we consider levels of wealth of zero, half the baseline, twice the baseline, and three times the baseline. We also consider the case of a non-saver at each age who by definition has zero wealth. For all cases, we consider the baseline density function with a constant hazard rate of reform.

Table 3 shows the results for the case of timing uncertainty over a benefit cut. Each panel of the



graph shows results for one of the five income groups considered in the previous results. The first column replicates the result for the 25 year old worker that begins with no assets for comparison. We find that additional initial wealth holdings reduce the welfare cost associated with timing uncertainty as the possible benefit cut has less impact on the overall level of consumption for the individual. Moreover, older individuals face larger costs associated with uncertain policy reforms. That is, as individuals are closer to retirement, the potential of a future benefit cut has greater costs for individuals as they are closer to retirement. This is the case even though the distribution of reform has the same exponential distribution as in the original results. The higher cost instead arises as retirement benefits become a larger portion of an individual's remaining lifetime income and the horizon to save more for retirement and hence mitigate the costs is shorter. Indeed, for a 65 year old with no accumulated assets the cost of uncertainty about the timing of reform is nearly 5% of remaining lifetime consumption. The costs are over 7.5% for a 65 year old non-saver.

Table 4 shows the results for the case of timing uncertainty over a tax increase. In this case, wealth still reduces the costs of uncertainty about the timing of reform, but older workers have lower costs as they have a shorter remaining working life that could be subject to the tax. To highlight this intuition, the welfare costs of 65 year old workers is exactly zero as they no longer face any uncertainty about their future income stream.

Finally, Table 5 shows the results for the case of double uncertainty about both the timing and structure of reform. Here, as in the initial results, the distribution of the structure of reform between a benefit cut and a tax increase is uniform. The pattern of results is similar to the case of benefit cuts with higher levels of wealth reducing the cost of uncertainty and older ages having larger welfare effects, but the magnitude of the welfare costs do not increase by as much for older individuals. A 65 year old with no wealth who saves optimally going forward has a welfare cost of nearly 3% of lifetime consumption while the non-saver is nearly 3.5%.

## 5.2. Grandfathering

Up to this point, we have assumed that the individual never escapes reform risk. However, almost all serious proposals to reform Social Security contain some element of grandfathering of older individuals into the old regime.<sup>21</sup> Here we assume the individual is exempt from the new regime and grandfathered into the old regime if he survives to age  $t_G \in (0, 1)$ . That is, after  $t_G$  the individual is exempt from any

---

<sup>21</sup>The only exception of which we are aware is the proposal to reduce the cost-of-living adjustments that Social Security beneficiaries receive by indexing benefits to the chained consumer price index.

reform.<sup>22</sup> We can continue to use the same notation and methods, as long as we set

$$\phi(t_1) = 0 \text{ for } t_1 \in [t_G, 1], \text{ and } \int_1^\infty \phi(t_1) dt_1 = 1 - \int_0^{t_G} \phi(t_1) dt_1. \quad (34)$$

We set  $t_G = 0.4$  to reflect grandfathering at age 55, which protects older workers from experiencing a benefit cut just prior to retirement. We ignore the fact that grandfathering would require the benefit cut experienced by younger individuals to be larger. Table 6 reports results when reform is grandfathered for the benefit cut, tax increase, and double uncertainty cases with the exponential distribution of reform dates. We find that grandfathering actually increases the cost of timing uncertainty relative to the results without grandfathering for both the benefit cut and tax reform cases. For double uncertainty, the welfare costs are higher for an optimal saver but slightly lower for non-savers. When interpreting these findings, recall that our measure of welfare only measures the cost of uncertainty about the timing and structure of reform, while netting out the wealth effects of the cost of reform itself. Therefore, individuals would still prefer to live in a world with grandfathering as it increases their expected lifetime wealth, but grandfathering actually increases the risks associated with the timing of reform because the individual either experiences a full cut to benefits or no cut, with no chance of anything in between these extremes.

This welfare measure is interesting for policy as it provides a fair comparison given the changes in costs of different reform options, since grandfathering any policy change would be costly. Note that in all cases, individuals know that reform will be grandfathered. While in reality almost all reform proposals would grandfather those close to retirement, it is not clear how well individuals understand this. In the 2012 wave of the Health and Retirement Study (HRS), a survey intended to be representative of older Americans, respondents were asked about the probability of their own current or future Social Security benefit being cut in the next 10 years. Among individuals aged 55 and older, the mean response to this question was 55 percent, and the 75th percentile response was 80%.

### 5.3. Extreme Political Risk

We have thus far used a uniform distribution over  $\alpha$  (the structural uncertainty parameter) in all of our calculations for the double uncertainty case. This assumption implies that any particular convex combination (compromise) between a full tax increase and a full benefit cut is just as likely as any other combination. This may be a reasonable starting assumption, but it could be the case that the structure of reform will ultimately look more like the outcome of a tug-of-war contest between two political parties,

---

<sup>22</sup>Actual reform proposals generally exempt individuals close to retirement from benefit cuts but not from tax increases.

with one side winning completely and the other side losing, rather than a compromise.<sup>23</sup>

In this subsection we assume that the structure of reform is uncertain and will be either all on the tax side or all on the benefit side, with no probability of a convex combination. We assume that these two possibilities are equally likely. This maximizes the degree of policy uncertainty along the structural dimension, which nearly doubles the welfare losses relative to the baseline with uniform structural uncertainty. This is true for both savers and non-savers. To see this, compare the results in Table 7 to those in Table 2.

#### 5.4. Larger Tax and Benefit Adjustments

The quantitative evidence from some theoretical macroeconomic models that include household behavioral responses and factor price responses to demographic and fiscal shocks suggests that the required tax and benefit adjustments could be much larger than what is estimated in the Trustees reports. Using the estimates of the required tax increase and benefit cut from Kitao (2014) as the base adjustment, together with the same penalty for delay that we have been using throughout, gives the new tax rate  $\tau_2(t_1) = \tau_1 + 6\% + 6.6\% \times t_1$  and the new benefit level  $b_2(t_1) = b_1 \times (1 - 33\% - 32\% \times t_1)$ . Table 8 shows how much larger the welfare costs of reform uncertainty can be under these larger tax and benefit adjustments.

#### 5.5. Differential Mortality

It is well known that low-income individuals suffer from lower survival probabilities at all ages relative to high-income individuals. We have recomputed our welfare analysis for the case of income-specific survival functions based on Social Security administrative data provided in Chart 1 of Waldron (2013). These data suggest that among 63 year old males, the death rate of the first, third, eighth and tenth lifetime income deciles are, respectively, 2.6, 1.3, 0.8, and 0.6 times that of the average of the fifth and sixth deciles.<sup>24</sup> Based on this, we assume that throughout their lives, maximum earners face age-specific hazard rates of dying that are 0.6 times those of average earners, high earners face age-specific hazard rates that are 0.8 times those of average earners, low earners face age-specific hazard rates that are 1.3 times those of average earners, and very low earners face age-specific hazard rates that are 2.6 times those

---

<sup>23</sup>See Baker et al. (2014) for a discussion of political polarization in recent years in American politics. Also see Davig and Foerster (2014) for a similar discussion.

<sup>24</sup>These relative death rates likely understate the extent of differential mortality as the sample in Waldron (2013) excludes disabled individuals, individuals who have not accumulated the 10 years of earnings required to qualify for Social Security, and individuals who do not survive to age 63.

of average earners. (If scaling by these factors causes an age-specific hazard rate to exceed 1, that hazard rate is set to 1).

Allowing hazard rates to differ by income groups causes almost no change to our calculations of the welfare costs of uncertainty about the timing of reform: the magnitude of the welfare cost is still about the same; the fact that uncertainty is regressive continues to hold; and non-savers continue to experience far bigger costs than savers.

Perhaps the most surprising result is that differential mortality does not undo the distributional effects. One may think that, because the regressivity of the uncertainty cost is driven by the progressivity of Social Security benefits, including differential mortality would undo our results since this would unwind (or even reverse) the progressivity of Social Security. While Coronado, Fullerton and Glass (1999) show that differential mortality unwinds the progressivity of Social Security in a cross-sectional sense, it does not unwind the progressivity in a longitudinal sense. At a moment in time, the ratio of aggregate benefits collected by survivors to aggregate taxes paid by workers would tend to be low for segments of the population with lower survival probabilities; but this has nothing to do with how a given worker treats Social Security taxes and benefits in an expected utility (longitudinal) model. In such a model, the progressivity of Social Security is only related to the benefit-earning rule and not to mortality risk, because the latter enters the model only through the discount factor in the utility function. Regardless of their survival type, expected utility maximizers make an optimal consumption-saving plan that accounts for the contingency that they survive until the maximum possible date. In other words, income flows (like Social Security) in the budget constraint of an expected utility maximizer are not discounted for survival risk.

## 5.6. Preference Parameters

Tables 9 and 10 provide alternative estimates of the welfare cost under different assumptions about the preference parameters. Not surprisingly, if we increase the degree of risk aversion, the cost of reform uncertainty can go up significantly. In Table 9 we report results for the case of  $\sigma = 5$  (recall that the baseline value was  $\sigma = 3$ ). And in Table 10 we report results for  $\rho = r = 2.9\%$  per year (recall that the baseline value was  $\rho = 0$ ).

## 6. Conclusion

The long-term insolvency of the Social Security system makes it difficult to plan and save for retirement because individuals do not know their future benefit levels or tax rates. This paper studies how policy uncertainty about the timing and structure of Social Security reform influences individual decision making and welfare. We solve and simulate optimal consumption/saving decisions in a setting with uncertainty about the timing and structure of a one-time Social Security reform shock. Optimal planning in the face of these two layers of policy uncertainty requires that households compute a contingent consumption/saving plan for every combination of the realization of the timing and structure of reform. Then households must recursively embed these contingent plans into an ex ante problem that assigns a continuation value to asset holdings based on the probabilities of the many contingencies.

We use our model to quantify the cost of policy uncertainty about the timing and structure of Social Security reform. We find that if individuals save optimally, then the welfare cost amounts to only a few *basis points* of total lifetime consumption. Optimal foresight/planning allows the individual to hedge away much of the welfare costs of policy uncertainty. However, at the other extreme, individuals who do not save experience welfare costs that can exceed 1% of their total lifetime consumption.

## References

- Abel, Andrew B., and Janice C. Eberly.** 2012. “Investment, Valuation, and Growth Options.” *Quarterly Journal of Finance*, 2(1).
- Amit, Raphael.** 1986. “Petroleum Reservoir Exploitation: Switching from Primary to Secondary Recovery.” *Operations Research*, 34(2): 534–549.
- Baker, Scott R., Nicholas Bloom, and Steven Davis.** 2013. “Measuring Economic Policy Uncertainty.” Stanford Working Paper.
- Baker, Scott R., Nicholas Bloom, Brandice Canes-Wrone, Steven J. Davis, and Jonathan Rodden.** 2014. “Why Has US Policy Uncertainty Risen Since 1960?” *American Economic Review: Papers and Proceedings*, 104(5): 56–60.
- Benítez-Silva, Hugo, Debra S. Dwyer, Frank Heiland, and Warren C. Sanderson.** 2007. “Retirement and Social Security Reform Expectations: A Solution to the New Early Retirement Puzzle.” Stony Brook Working Paper.
- Bi, Huixin, Eric M. Leeper, and Campbell Leith.** 2012. “Uncertain Fiscal Consolidations.” Bank of Canada Working Paper.
- Blundell, Richard, and Thomas M. Stoker.** 1999. “Consumption and the Timing of Income Risk.” *European Economic Review*, 43: 475–507.
- Boucekkine, Raouf, Aude Pommeret, and Fabien Prieur.** 2012. “Optimal Regime Switching and Threshold Effects: Theory and Application to a Resource Extraction Problem under Irreversibility.” LAMETA, Universtiy of Montpellier Working Papers.
- Boucekkine, Raouf, Aude Pommeret, and Fabien Prieur.** 2013a. “On the timing and optimality of capital controls: Public expenditures, debt dynamics and welfare.” *International Journal of Economic Theory*, 9(1): 101–112.
- Boucekkine, Raouf, Aude Pommeret, and Fabien Prieur.** 2013b. “Technological vs. Ecological Switch and the Environmental Kuznets Curve.” *American Journal of Agricultural Economics*, 95(2): 252–260.
- Boucekkine, Raouf, Cagri Saglam, and Thomas Vallee.** 2004. “Technology Adoption Under Embodiment: A Two-Stage Optimal Control Approach.” *Macroeconomic Dynamics*, 8(02): 250–271.

- Bütler, Monika.** 1999. “Anticipation Effects of Looming Public-Pension Reforms.” *Carnegie-Rochester Conference Series on Public Policy*, 50(1): 119–159.
- Clarke, Harry R., and William J. Reed.** 1994. “Consumption/Pollution Tradeoffs in an Environment Vulnerable to Pollution-Related Catastrophic Collapse.” *Journal of Economic Dynamics and Control*, 18: 991–1010.
- Coronado, Julia Lynn, Don Fullerton, and Thomas Glass.** 1999. “Distributional Impacts of Proposed Changes to the Social Security System.” National Bureau of Economic Research NBER Working Paper.
- Dasgupta, Partha, and Geoffrey Heal.** 1974. “The Optimal Depletion of Exhaustible Resources.” *Review of Economic Studies*, 41: 3–28.
- Davig, Troy, and Andrew Foerster.** 2014. “Uncertainty and Fiscal Cliffs.” Federal Reserve Bank of Kansas City Working Paper.
- Davig, Troy, Eric M. Leeper, and Todd B. Walker.** 2010. “Unfunded Liabilities and Uncertain Fiscal Financing.” *Journal of Monetary Economics*, 57(5): 600–619.
- Diamond, Peter, and Jonathan Gruber.** 1999. “Social Security and Retirement in the United States.” National Bureau of Economic Research NBER Working Paper.
- Dogan, Erol, Cuong Le Van, and Cagri Saglam.** 2011. “Optimal Timing of Regime Switching in Optimal Growth Models: A Sobolev Space Approach.” *Mathematical Social Sciences*, 61: 97–103.
- Dominitz, Jeff, Charles F. Manski, and Jordan Heinz.** 2003. “Will Social Security be There for You?: How Americans Perceive their Benefits.” National Bureau of Economic Research, Inc NBER Working Papers 9798.
- Eeckhoudt, Louis, Christian Gollier, and Nicolas Treich.** 2005. “Optimal Consumption and the Timing of the Resolution of Uncertainty.” *European Economic Review*, 49: 761–773.
- Epstein, Larry G., Emmanuel Farhi, and Tomasz Strzalecki.** 2014. “How Much Would You Pay to Resolve Long-Run Risk.” *American Economic Review*, forthcoming.
- Evans, Richard W., Laurence J. Kotlikoff, and Kerk L. Phillips.** 2012. “Game Over: Simulating Unsustainable Fiscal Policy.” National Bureau of Economic Research, Inc NBER Working Papers 17917.

- Fernández-Villaverde, Jesús, Pablo Guerrón-Quintana, Keith Kuester, and Juan Rubio-Ramírez.** 2013. “Fiscal Volatility Shocks and Economic Activity.” Working Paper.
- Gokhale, Jagadeesh.** 2013. “Social Security Reform: Does Privatization Still Make Sense?” *Harvard Journal on Legislation*, 50: 169–207.
- Gomes, Francisco J., Laurence J. Kotlikoff, and Luis M. Viceira.** 2007. “The Excess Burden of Government Indecision.” National Bureau of Economic Research, Inc NBER Working Papers 12859.
- Gourinchas, Pierre-Olivier, and Jonathan A. Parker.** 2002. “Consumption Over the Life Cycle.” *Econometrica*, 70(1): 47–89.
- Hoel, Michael.** 1978. “Resource Extraction when a Future Substitute has an Uncertain Cost.” *Review of Economic Studies*, 45(3): 637–644.
- Hugonnier, Julien, Florian Pelgrin, and Aude Pommeret.** 2006. “Technology Adoption under Uncertainty in General Equilibrium.” Working Paper.
- Hurst, Erik E.** 2006. “Grasshoppers, Ants and Pre-Retirement Wealth: A Test of Permanent Income Consumers.” University of Chicago Working Paper.
- Kamien, Morton I., and Nancy L. Schwartz.** 1971. “Optimal Maintenance and Sale Age for a Machine Subject to Failure.” *Management Science*, 17(8): 495–504.
- Kemp, Murray C., and Ngo Van Long.** 1977. “Optimal Control Problems with Integrands Discontinuous with Respect to Time.” *The Economic Record*, 53(142&143): 405–20.
- Kitao, Sagiri.** 2014. “Sustainable Social Security: Four Options.” *Review of Economic Dynamics*, 17(4): 756–779.
- Laibson, David I.** 1997. “Golden Eggs and Hyperbolic Discounting.” *Quarterly Journal of Economics*, 112(2): 443–478.
- Liebman, Jeffrey B., and Erzo F.P. Luttmer.** 2014. “Would People Behave Differently If They Better Understood Social Security? Evidence From a Field Experiment.” *American Economic Journal: Economic Policy*, forthcoming.
- Luttmer, Erzo F. P., and Andrew A. Samwick.** 2012. “The Welfare Cost of Perceived Policy Uncertainty: Evidence from Social Security.” Dartmouth College Working Paper.



- Makris, Miltiadis.** 2001. “Necessary conditions for infinite-horizon discounted two-stage optimal control problems.” *Journal of Economic Dynamics and Control*, 25(12): 1935–1950.
- Mangasarian, Olvi L.** 1966. “Sufficient Conditions for the Optimal Control of Nonlinear Systems.” *SIAM Journal on Control*, 4(1): 139–152.
- Manski, Charles F.** 2004. “Measuring Expectations.” *Econometrica*, 72(5): 1329–1376.
- McGrattan, Ellen R., and Edward C. Prescott.** 2014. “On Financing Retirement with an Aging Population.” University of Minnesota Working Paper.
- Pommeret, Aude, and Katheline Schubert.** 2009. “Abatement Technology Adoption Under Uncertainty.” *Macroeconomic Dynamics*, 13(04): 493–522.
- Saglam, Cagri.** 2011. “Optimal Pattern of Technology Adoptions under Embodiment: A Multi-Stage Optimal Control Approach.” *Optimal Control Applications and Methods*, 32: 574–586.
- Sargent, Thomas.** 2005. “Ambiguity in American Monetary and Fiscal Policy.” New York University Working Paper.
- Stokey, Nancy L.** 2014. “Wait-and-See: Investment Options under Policy Uncertainty.” University of Chicago Working Paper.
- Tahvonen, Olli, and Cees Withagen.** 1996. “Optimality of Irreversible Pollution Accumulation.” *Journal of Economic Dynamics and Control*, 20: 1775–1795.
- Thaler, Richard H.** 1994. “Psychology and Savings Policies.” *American Economic Review*, 84(2): 186–192.
- Tomiyama, Ken.** 1985. “Two-stage optimal control problems and optimality conditions.” *Journal of Economic Dynamics and Control*, 9(3): 317–337.
- van der Wiel, Karen.** 2008. “Preparing for Policy Changes: Social Security Expectations and Pension Scheme Participation.” Institute for the Study of Labor IZA DP No. 3623.
- Waldron, Hilary.** 2013. “Mortality Differentials by Lifetime Earnings Decile: Implications for Evaluations of Proposed Social Security Law Changes.” *Social Security Bulletin*, 73(1).
- Wright, Ian, Nick Bloom, and Jose Barrero.** 2014. “Investment and the Term Structure of Uncertainty.” Stanford University Working Paper.

# Appendices

## Appendix A: Proof of Necessary Conditions

### Step 1. Solve the post-reform ( $t = t_1$ ) subproblem:

This step of the backward induction procedure is a standard Pontryagin problem. The optimal control and state paths after the switch is realized must solve a standard deterministic control problem. We denote the solution to this subproblem  $(u_2^*(t|t_1, x(t_1)), x_2^*(t|t_1, x(t_1)))_{t \in [t_1, T]}$ , where the extra notation is meant to convey the dependence of the solution on the switch date  $t_1$  and on the state variable at that date  $x(t_1)$ . The last part of Step 1 is strictly for convenience: we take this solution and change the time dummy  $t$  to  $z$ , and change the switch point  $t_1$  to  $t$  and write  $(u_2^*(z|t, x(t)), x_2^*(z|t, x(t)))_{z \in [t, T]}$ . Thus we have the optimal control and state paths for all points in time  $z$  greater than switch point  $t$ . This change of dummies is innocuous but proves to be very helpful below in the process of linking the subproblems together through the continuation functional in the next step.

### Step 2. Solve the pre-reform ( $t = 0$ ) subproblem:

This step requires a little more explanation. The purpose of this step is to find the optimal paths for the control and state before the realization of the switch. Hence, using the solution from Step 1, the objective functional is

$$\max_{u(t)_{t \in [0, T]}} : J_1 = \mathbb{E} \left[ \int_0^{t_1} f_1(t, u(t), x(t)) dt + \int_{t_1}^T f_2(t, u_2^*(t|t_1, x(t_1)), x_2^*(t|t_1, x(t_1))) dt \right] \quad (\text{A1})$$

or

$$\begin{aligned} \max_{u(t)_{t \in [0, T]}} : J_1 = & \int_0^T \int_0^{t_1} \phi(t_1) f_1(t, u(t), x(t)) dt dt_1 + \int_0^T \phi(t_1) \mathcal{S}(t_1, x(t_1)) dt_1 \\ & + \int_T^\infty \int_0^T \phi(t_1) f_1(t, u(t), x(t)) dt dt_1, \end{aligned} \quad (\text{A2})$$

where  $\mathcal{S}(t_1, x(t_1))$  is the continuation value or continuation function,

$$\mathcal{S}(t_1, x(t_1)) = \int_{t_1}^T f_2(t, u_2^*(t|t_1, x(t_1)), x_2^*(t|t_1, x(t_1))) dt, \quad (\text{A3})$$

and likewise  $\int_0^T \phi(t_1) \mathcal{S}(t_1, x(t_1)) dt_1$  is the continuation **functional**. The constraints are

$$\frac{dx(t)}{dt} = g_1(t, u(t), x(t)), \text{ for } t \in [0, T], \quad (\text{A4})$$

$$x(0) = x_0, x(T) = x_T. \quad (\text{A5})$$

Note that the control and state variables are defined over the entire planning interval because the switch could happen at any time on this interval, and hence the pre-switch problem amounts to choosing a path for these variables over the full interval. The marginal valuation of the state variable reflects the fact that the switch could happen at any moment in time, and hence the entire timepath of the state variable is relevant in determining the continuation value during the second stage. The solution to the above problem is the one that will be followed up to the random switch point.

This appears to be a non-standard control problem, but with some algebra we can convert it into a standard one for which the standard Maximum Principle applies. To make progress, change the dummy of integration in the second integral in  $J_1$  from  $t_1$  to  $t$ , and also change the dummy in the continuation functional from  $t$  to  $z$ . Now we restate the objective functional as

$$\begin{aligned} \max_{u(t)_{t \in [0, T]}} : J_1 = & \int_0^T \int_0^{t_1} \phi(t_1) f_1(t, u(t), x(t)) dt dt_1 + \int_0^T \phi(t) \mathcal{S}(t, x(t)) dt \\ & + \int_T^\infty \int_0^T \phi(t_1) f_1(t, u(t), x(t)) dt dt_1, \end{aligned} \quad (\text{A6})$$

where

$$\mathcal{S}(t, x(t)) = \int_t^T f_2(z, u_2^*(z|t, x(t)), x_2^*(z|t, x(t))) dz. \quad (\text{A7})$$

This innocuous change of variables is helpful because it allows us to write the continuation function  $\mathcal{S}$  as a function of  $x(t)$ . In doing so,  $u_2^*(z|t, x(t))$  and  $x_2^*(z|t, x(t))$  now take on the interpretation of the optimal control and state variables for all points in time  $z$  that are beyond the switch date  $t$  and conditional on  $x(t)$ .

Changing the order of integration in the first term in  $J_1$ , along with applying Fubini's Theorem to the third term in  $J_1$ , causes a more manageable Pontryagin problem to emerge. Let us now restate our problem one last time

$$\max_{u(t)_{t \in [0, T]}} : J_1 = \int_0^T \left\{ \left[ \int_t^\infty \phi(t_1) dt_1 \right] f_1(t, u(t), x(t)) + \phi(t) \mathcal{S}(t, x(t)) \right\} dt, \quad (\text{A8})$$

subject to

$$\mathcal{S}(t, x(t)) = \int_t^T f_2(z, u_2^*(z|t, x(t)), x_2^*(z|t, x(t))) dz, \quad (\text{A9})$$

$$\frac{dx(t)}{dt} = g_1(t, u(t), x(t)), \text{ for } t \in [0, T], \quad (\text{A10})$$

$$x(0) = x_0, \quad x(T) = x_T. \quad (\text{A11})$$

This reformulated objective functional has an intuitive interpretation. The first term in the integrand gives the payoff of  $u(t)$  and  $x(t)$  through the function  $f_1(t, u(t), x(t))$ , weighted by the probability that the random switch will occur sometime after  $t$ . The second term gives the payoff of holding  $x(t)$  through the continuation value  $\mathcal{S}(t, x(t))$ , weighted by the density function  $\phi(t)$ . Thus, the first payoff term  $f_1$  is weighted by one minus the c.d.f. because this payoff is relevant as long as the switch comes later than  $t$ , whereas the second payoff term  $\mathcal{S}$  is weighted by the p.d.f. because this payoff is relevant only at the switch point. The standard Maximum Principle can be applied to this reformulated subproblem. We denote the solution to this subproblem  $(u_1^*(t), x_1^*(t))_{t \in [0, T]}$ . This is the solution path for all  $t$  before the realization of the random switch.

## Appendix B. Derivation of Solution to Application 1

Using our solution method as a guide, we can solve the individual's problem recursively.

### Step 1. Solve the post-reform ( $t = t_1$ ) subproblem:

We first solve the Pontryagin subproblem corresponding to the moment that reform occurs. This is a deterministic, fixed endpoint control problem

$$\max_{c(t)_{t \in [t_1, T]}} : J_2 = \int_{t_1}^T e^{-\rho t} \Psi(t) \frac{c(t)^{1-\sigma}}{1-\sigma} dt, \quad (\text{B1})$$

subject to

$$\frac{dk(t)}{dt} = rk(t) + y_2(t|t_1) - c(t), \quad \text{for } t \in [t_1, T], \quad (\text{B2})$$

$$t_1 \text{ given, } k(t_1) \text{ given, } k(T) = 0. \quad (\text{B3})$$

Form the Hamiltonian  $\mathcal{H}_2$  with multiplier  $\lambda_2(t)_{t \in [t_1, T]}$  and compute the necessary conditions

$$\mathcal{H}_2 = e^{-\rho t} \Psi(t) \frac{c(t)^{1-\sigma}}{1-\sigma} + \lambda_2(t) [rk(t) + y_2(t|t_1) - c(t)], \quad (\text{B4})$$

$$\frac{\partial \mathcal{H}_2}{\partial c(t)} = e^{-\rho t} \Psi(t) c(t)^{-\sigma} - \lambda_2(t) = 0, \quad \text{for } t \in [t_1, T], \quad (\text{B5})$$

$$\frac{d\lambda_2(t)}{dt} = -\frac{\partial \mathcal{H}_2}{\partial k(t)} = -r\lambda_2(t), \quad \text{for } t \in [t_1, T]. \quad (\text{B6})$$

Rewrite the costate equation

$$\lambda_2(t) = \lambda_2(t_1) e^{-r(t-t_1)}, \quad (\text{B7})$$

and collapse the necessary conditions into a single equation

$$e^{-\rho t} \Psi(t) c(t)^{-\sigma} = \lambda_2(t_1) e^{-r(t-t_1)}. \quad (\text{B8})$$

Solve for  $c(t)$

$$c(t) = \lambda_2(t_1)^{-1/\sigma} e^{[(r-\rho)t-r t_1]/\sigma} \Psi(t)^{1/\sigma}. \quad (\text{B9})$$

Solve differential equation (B2) using the boundary conditions in (B3)

$$k(t_1) + \int_{t_1}^T y_2(v|t_1) e^{-r(v-t_1)} dv = \int_{t_1}^T c(v) e^{-r(v-t_1)} dv. \quad (\text{B10})$$

Insert (B9) into (B10)

$$k(t_1) + \int_{t_1}^T y_2(v|t_1) e^{-r(v-t_1)} dv = \int_{t_1}^T \lambda_2(t_1)^{-1/\sigma} e^{-r(v-t_1)+[(r-\rho)v-r t_1]/\sigma} \Psi(v)^{1/\sigma} dv, \quad (\text{B11})$$

and solve for the constant

$$\lambda_2(t_1)^{-1/\sigma} = \frac{k(t_1) + \int_{t_1}^T y_2(v|t_1) e^{-r(v-t_1)} dv}{\int_{t_1}^T e^{-r(v-t_1)+[(r-\rho)v-r t_1]/\sigma} \Psi(v)^{1/\sigma} dv}. \quad (\text{B12})$$

Insert this into (B9) to obtain the solution consumption path

$$c_2^*(t|t_1, k(t_1)) = \frac{k(t_1) + \int_{t_1}^T e^{-r(v-t_1)} y_2(v|t_1) dv}{\int_{t_1}^T e^{-r(v-t_1)+(r-\rho)v/\sigma} \Psi(v)^{1/\sigma} dv} e^{(r-\rho)t/\sigma} \Psi(t)^{1/\sigma}, \text{ for } t \in [t_1, T]. \quad (\text{B13})$$

This is the optimal consumption path after the reform shock has hit.

Anticipating our method for solving the pre-reform subproblem, we will need to change the time dummies: now think of  $t$  as the reform date and  $z$  as any time after the reform date. Thus, rewrite the solution as

$$c_2^*(z|t, k(t)) = \frac{k(t) + \int_t^T e^{-r(v-t)} y_2(v|t) dv}{\int_t^T e^{-r(v-t)+(r-\rho)v/\sigma} \Psi(v)^{1/\sigma} dv} e^{(r-\rho)z/\sigma} \Psi(z)^{1/\sigma}, \text{ for } z \in [t, T]. \quad (\text{B14})$$

## Step 2. Solve the pre-reform ( $t = 0$ ) subproblem:

Next we solve the  $t = 0$  subproblem,

$$\max_{c(t)_{t \in [0, T]}} : J_1 = \int_0^T \left\{ \left[ \int_t^\infty \phi(t_1) dt_1 \right] e^{-\rho t} \Psi(t) \frac{c(t)^{1-\sigma}}{1-\sigma} + \phi(t) \mathcal{S}(t, k(t)) \right\} dt, \quad (\text{B15})$$

subject to

$$\begin{aligned}\mathcal{S}(t, k(t)) &= \int_t^T e^{-\rho z} \Psi(z) \frac{c_2^*(z|t, k(t))^{1-\sigma}}{1-\sigma} dz \\ &= \frac{1}{1-\sigma} \left[ \frac{k(t) + \int_t^T e^{-r(v-t)} y_2(v|t) dv}{\int_t^T e^{-r(v-t)+(r-\rho)v/\sigma} \Psi(v)^{1/\sigma} dv} \right]^{1-\sigma} \int_t^T e^{[r(1-\sigma)-\rho]z/\sigma} \Psi(z)^{1/\sigma} dz,\end{aligned}\quad (\text{B16})$$

$$\frac{dk(t)}{dt} = rk(t) + y_1(t) - c(t), \text{ for } t \in [0, T], \quad (\text{B17})$$

$$k(0) = 0, k(T) = 0. \quad (\text{B18})$$

Form the Hamiltonian  $\mathcal{H}_1$  with multiplier  $\lambda_1(t)_{t \in [0, T]}$  and compute the necessary conditions<sup>25</sup>

$$\mathcal{H}_1 = \left[ \int_t^\infty \phi(t_1) dt_1 \right] e^{-\rho t} \Psi(t) \frac{c(t)^{1-\sigma}}{1-\sigma} + \phi(t) \mathcal{S}(t, k(t)) + \lambda_1(t) [rk(t) + y_1(t) - c(t)], \quad (\text{B19})$$

$$\frac{\partial \mathcal{H}_1}{\partial c(t)} = \left[ \int_t^\infty \phi(t_1) dt_1 \right] e^{-\rho t} \Psi(t) c(t)^{-\sigma} - \lambda_1(t) = 0, \text{ for } t \in [0, T], \quad (\text{B20})$$

$$\frac{d\lambda_1(t)}{dt} = -\frac{\partial \mathcal{H}_1}{\partial k(t)} = -\phi(t) \left[ \frac{k(t) + \int_t^T e^{-r(v-t)} y_2(v|t) dv}{\int_t^T e^{-r(v-t)+(r-\rho)v/\sigma} \Psi(v)^{1/\sigma} dv} \right]^{-\sigma} e^{-rt} - r\lambda_1(t), \text{ for } t \in [0, T]. \quad (\text{B21})$$

Differentiate (B20) with respect to  $t$

$$\begin{aligned}0 &= -\phi(t) e^{-\rho t} \Psi(t) c(t)^{-\sigma} + \left[ \int_t^\infty \phi(t_1) dt_1 \right] \left[ \frac{d\Psi(t)}{dt} e^{-\rho t} - \rho \Psi(t) e^{-\rho t} \right] c(t)^{-\sigma} \\ &\quad - \left[ \int_t^\infty \phi(t_1) dt_1 \right] \sigma e^{-\rho t} \Psi(t) c(t)^{-\sigma-1} \frac{dc(t)}{dt} - \frac{d\lambda_1(t)}{dt}.\end{aligned}\quad (\text{B22})$$

Insert (B20) into (B21)

$$\frac{d\lambda_1(t)}{dt} = -\phi(t) \left[ \frac{k(t) + \int_t^T e^{-r(v-t)} y_2(v|t) dv}{\int_t^T e^{-r(v-t)+(r-\rho)v/\sigma} \Psi(v)^{1/\sigma} dv} \right]^{-\sigma} e^{-rt} - r \left[ \int_t^\infty \phi(t_1) dt_1 \right] e^{-\rho t} \Psi(t) c(t)^{-\sigma}, \quad (\text{B23})$$

and then insert (B23) into (B22) to obtain the Euler equation

$$\begin{aligned}\frac{dc(t)}{dt} &= \left( \frac{c(t)^{\sigma+1}}{\Psi(t)} \left[ \frac{k(t) + \int_t^T e^{-r(v-t)} y_2(v|t) dv}{\int_t^T e^{-r(v-t)+(r-\rho)v/\sigma} \Psi(v)^{1/\sigma} dv} \right]^{-\sigma} e^{(\rho-r)t} - c(t) \right) \times \left[ \frac{\sigma}{\phi(t)} \int_t^\infty \phi(t_1) dt_1 \right]^{-1} \\ &\quad + \left[ \frac{d\Psi(t)}{dt} \frac{1}{\Psi(t)} + r - \rho \right] \frac{c(t)}{\sigma}.\end{aligned}\quad (\text{B24})$$

<sup>25</sup>The necessary conditions are also sufficient because the integrand of  $J_1$  is concave in  $c(t)$  and  $k(t)$ .

---



---

**Table 1. Welfare Loss from Uncertainty about the Timing of Reform:**

---

*Panel A. Baseline Density Function with Constant Hazard Rate of Reform*

---

Income	Rep.	Benefit Reform		Tax Reform	
	Rate	saver	non-saver	saver	non-saver
very low	67.5%	0.06%	0.82%	0.02%	0.03%
low	49.0%	0.04%	1.01%	0.02%	0.02%
average	36.4%	0.02%	1.15%	0.02%	0.01%
high	30.1%	0.02%	1.22%	0.02%	0.01%
max	24.0%	0.01%	1.27%	0.03%	0.01%

---

*Panel B. Alternative Density Function with Mode at 2034*

---

Income	Rep.	Benefit Reform		Tax Reform	
	Rate	saver	non-saver	saver	non-saver
very low	67.5%	0.04%	0.80%	0.02%	0.03%
low	49.0%	0.02%	0.98%	0.02%	0.02%
average	36.4%	0.01%	1.11%	0.02%	0.02%
high	30.1%	0.01%	1.16%	0.03%	0.01%
max	24.0%	0.01%	1.21%	0.03%	0.01%

---

\*All welfare costs are expressed as a percentage of total lifetime consumption.

---

---



---

**Table 2. Welfare Loss from Uncertainty about Timing & Structure of Reform:**

---

*Panel A. Baseline Density Function with Constant Hazard Rate of Reform*

---

Income	Rep. Rate	Benefit Reform		Tax Reform		Double Uncertainty	
		saver	non-saver	saver	non-saver	saver	non-saver
		very low	67.5%	0.06%	0.82%	0.02%	0.03%
low	49.0%	0.04%	1.01%	0.02%	0.02%	0.03%	1.04%
average	36.4%	0.02%	1.15%	0.02%	0.01%	0.02%	1.23%
high	30.1%	0.02%	1.22%	0.02%	0.01%	0.02%	1.32%
max	24.0%	0.01%	1.27%	0.03%	0.01%	0.01%	1.40%

---

*Panel B. Alternative Density Function with Mode at 2034*

---

Income	Rep. Rate	Benefit Reform		Tax Reform		Double Uncertainty	
		saver	non-saver	saver	non-saver	saver	non-saver
		very low	67.5%	0.04%	0.80%	0.02%	0.03%
low	49.0%	0.02%	0.98%	0.02%	0.02%	0.05%	1.34%
average	36.4%	0.01%	1.11%	0.02%	0.02%	0.03%	1.57%
high	30.1%	0.01%	1.16%	0.03%	0.01%	0.02%	1.68%
max	24.0%	0.01%	1.21%	0.03%	0.01%	0.02%	1.78%

---

\*All welfare costs are expressed as a percentage of total lifetime consumption.

---



---

---

**Table 3. Welfare Loss by Age from Timing Uncertainty about Benefit Reform:**

Panel A. Very Low Income Group (67.5% Rep. Rate)

Assets	Age 25	Age 45	Age 55	Age 65
0	—	0.43%	1.52%	4.74%
$0.5k_1^*(t_0)$	—	0.39%	1.25%	3.59%
$k_1^*(t_0)$	0.06%	0.35%	1.05%	2.82%
$2k_1^*(t_0)$	—	0.29%	0.77%	1.87%
$3k_1^*(t_0)$	—	0.25%	0.59%	1.33%
non-saver	0.82%	2.60%	4.87%	7.55%

Panel B. Low Income Group (49.0% Rep. Rate)

Assets	Age 25	Age 45	Age 55	Age 65
0	—	0.28%	1.13%	4.74%
$0.5k_1^*(t_0)$	—	0.25%	0.86%	3.00%
$k_1^*(t_0)$	0.04%	0.22%	0.68%	2.08%
$2k_1^*(t_0)$	—	0.17%	0.45%	1.17%
$3k_1^*(t_0)$	—	0.14%	0.32%	0.75%
non-saver	1.01%	2.90%	5.19%	7.55%

Panel C. Average Income Group (36.4% Rep. Rate)

Assets	Age 25	Age 45	Age 55	Age 65
0	—	0.19%	0.82%	4.74%
$0.5k_1^*(t_0)$	—	0.16%	0.59%	2.45%
$k_1^*(t_0)$	0.02%	0.14%	0.44%	1.50%
$2k_1^*(t_0)$	—	0.10%	0.27%	0.73%
$3k_1^*(t_0)$	—	0.08%	0.18%	0.43%
non-saver	1.15%	3.07%	5.36%	7.55%

Panel D. High Income Group (30.1% Rep. Rate)

Assets	Age 25	Age 45	Age 55	Age 65
0	—	0.14%	0.66%	4.74%
$0.5k_1^*(t_0)$	—	0.12%	0.45%	2.11%
$k_1^*(t_0)$	0.02%	0.10%	0.32%	1.19%
$2k_1^*(t_0)$	—	0.07%	0.19%	0.54%
$3k_1^*(t_0)$	—	0.06%	0.13%	0.30%
non-saver	1.22%	3.15%	5.43%	7.55%

Panel E. Max Income Group (24.0% Rep. Rate)

Assets	Age 25	Age 45	Age 55	Age 65
0	—	0.10%	0.49%	4.74%
$0.5k_1^*(t_0)$	—	0.08%	0.32%	1.72%
$k_1^*(t_0)$	0.01%	0.07%	0.22%	0.89%
$2k_1^*(t_0)$	—	0.05%	0.13%	0.36%
$3k_1^*(t_0)$	—	0.04%	0.08%	0.20%
non-saver	1.27%	3.21%	5.49%	7.55%

---

\*All welfare costs are expressed as a percentage of total lifetime consumption.

---

---



---

**Table 4. Welfare Loss by Age from Timing Uncertainty about Tax Reform:**

Panel A. Very Low Income Group (67.5% Rep. Rate)

Assets	Age 25	Age 45	Age 55	Age 65
0	—	0.03%	0.01%	0.00%
$0.5k_1^*(t_0)$	—	0.02%	0.01%	0.00%
$k_1^*(t_0)$	0.02%	0.02%	0.01%	0.00%
$2k_1^*(t_0)$	—	0.01%	0.00%	0.00%
$3k_1^*(t_0)$	—	0.01%	0.00%	0.00%
non-saver	0.03%	0.04%	0.03%	0.00%

Panel B. Low Income Group (49.0% Rep. Rate)

Assets	Age 25	Age 45	Age 55	Age 65
0	—	0.03%	0.02%	0.00%
$0.5k_1^*(t_0)$	—	0.03%	0.01%	0.00%
$k_1^*(t_0)$	0.02%	0.03%	0.01%	0.00%
$2k_1^*(t_0)$	—	0.02%	0.01%	0.00%
$3k_1^*(t_0)$	—	0.02%	0.00%	0.00%
non-saver	0.02%	0.03%	0.02%	0.00%

Panel C. Average Income Group (36.4% Rep. Rate)

Assets	Age 25	Age 45	Age 55	Age 65
0	—	0.04%	0.02%	0.00%
$0.5k_1^*(t_0)$	—	0.03%	0.01%	0.00%
$k_1^*(t_0)$	0.02%	0.03%	0.01%	0.00%
$2k_1^*(t_0)$	—	0.02%	0.01%	0.00%
$3k_1^*(t_0)$	—	0.02%	0.00%	0.00%
non-saver	0.01%	0.02%	0.01%	0.00%

Panel D. High Income Group (30.1% Rep. Rate)

Assets	Age 25	Age 45	Age 55	Age 65
0	—	0.04%	0.02%	0.00%
$0.5k_1^*(t_0)$	—	0.04%	0.02%	0.00%
$k_1^*(t_0)$	0.02%	0.03%	0.01%	0.00%
$2k_1^*(t_0)$	—	0.02%	0.00%	0.00%
$3k_1^*(t_0)$	—	0.02%	0.00%	0.00%
non-saver	0.01%	0.01%	0.01%	0.00%

Panel E. Max Income Group (24.0% Rep. Rate)

Assets	Age 25	Age 45	Age 55	Age 65
0	—	0.05%	0.03%	0.00%
$0.5k_1^*(t_0)$	—	0.04%	0.02%	0.00%
$k_1^*(t_0)$	0.03%	0.03%	0.01%	0.00%
$2k_1^*(t_0)$	—	0.02%	0.00%	0.00%
$3k_1^*(t_0)$	—	0.02%	0.00%	0.00%
non-saver	0.01%	0.01%	0.01%	0.00%

\*All welfare costs are expressed as a percentage of total lifetime consumption.

---

---



---

**Table 5. Welfare Loss by Age from Timing and Structural Uncertainty:**

Panel A. Very Low Income Group (67.5% Rep. Rate)

Assets	Age 25	Age 45	Age 55	Age 65
0	—	0.42%	1.22%	2.93%
$0.5k_1^*(t_0)$	—	0.36%	0.93%	1.96%
$k_1^*(t_0)$	0.06%	0.32%	0.73%	1.41%
$2k_1^*(t_0)$	—	0.25%	0.48%	0.83%
$3k_1^*(t_0)$	—	0.20%	0.34%	0.54%
non-saver	0.81%	1.91%	2.75%	3.46%

Panel B. Low Income Group (49.0% Rep. Rate)

Assets	Age 25	Age 45	Age 55	Age 65
0	—	0.27%	0.89%	2.93%
$0.5k_1^*(t_0)$	—	0.23%	0.65%	1.72%
$k_1^*(t_0)$	0.03%	0.19%	0.49%	1.13%
$2k_1^*(t_0)$	—	0.15%	0.31%	0.60%
$3k_1^*(t_0)$	—	0.12%	0.21%	0.37%
non-saver	1.04%	2.21%	3.01%	3.46%

Panel C. Average Income Group (36.4% Rep. Rate)

Assets	Age 25	Age 45	Age 55	Age 65
0	—	0.17%	0.63%	2.93%
$0.5k_1^*(t_0)$	—	0.14%	0.43%	1.42%
$k_1^*(t_0)$	0.02%	0.12%	0.31%	0.84%
$2k_1^*(t_0)$	—	0.09%	0.19%	0.39%
$3k_1^*(t_0)$	—	0.07%	0.12%	0.23%
non-saver	1.23%	2.40%	3.16%	3.46%

Panel D. High Income Group (30.1% Rep. Rate)

Assets	Age 25	Age 45	Age 55	Age 65
0	—	0.13%	0.50%	2.93%
$0.5k_1^*(t_0)$	—	0.10%	0.33%	1.26%
$k_1^*(t_0)$	0.02%	0.09%	0.24%	0.70%
$2k_1^*(t_0)$	—	0.06%	0.14%	0.31%
$3k_1^*(t_0)$	—	0.05%	0.09%	0.17%
non-saver	1.32%	2.48%	3.22%	3.46%

Panel E. Max Income Group (24.0% Rep. Rate)

Assets	Age 25	Age 45	Age 55	Age 65
0	—	0.09%	0.37%	2.93%
$0.5k_1^*(t_0)$	—	0.07%	0.24%	1.07%
$k_1^*(t_0)$	0.01%	0.06%	0.17%	0.55%
$2k_1^*(t_0)$	—	0.04%	0.09%	0.22%
$3k_1^*(t_0)$	—	0.03%	0.06%	0.12%
non-saver	1.40%	2.55%	3.27%	3.46%

\*All welfare costs are expressed as a percentage of total lifetime consumption.

---

---

---

**Table 6. Welfare Loss under Grandfathering at Age 55:**

Income	Rep. Rate	Benefit Reform		Tax Reform		Double Uncertainty	
		saver	non-saver	saver	non-saver	saver	non-saver
		very low	67.5%	0.11%	1.08%	0.02%	0.03%
low	49.0%	0.07%	1.36%	0.02%	0.02%	0.04%	0.91%
average	36.4%	0.04%	1.56%	0.03%	0.01%	0.03%	1.08%
high	30.1%	0.03%	1.66%	0.03%	0.01%	0.03%	1.16%
max	24.0%	0.02%	1.74%	0.03%	0.01%	0.02%	1.24%

\*All welfare costs are expressed as a percentage of total lifetime consumption.

---

---

---

**Table 7. Welfare Loss with Extreme Political Risk:**

Income	Rep. Rate	Double Uncertainty	
		saver	non-saver
		very low	67.5%
low	49.0%	0.08%	2.78%
average	36.4%	0.04%	3.25%
high	30.1%	0.03%	3.48%
max	24.0%	0.03%	3.69%

\*All welfare costs are expressed as a percentage of total lifetime consumption.

---

---

---

**Table 8. Welfare Loss under Severe Estimates of Structural Reform:**

Income	Rep. Rate	Benefit Reform		Tax Reform		Double Uncertainty	
		saver	non-saver	saver	non-saver	saver	non-saver
		very low	67.5%	0.10%	1.71%	0.06%	0.09%
low	49.0%	0.06%	2.03%	0.07%	0.06%	0.07%	2.47%
average	36.4%	0.04%	2.23%	0.08%	0.04%	0.05%	2.86%
high	30.1%	0.03%	2.32%	0.08%	0.03%	0.04%	3.04%
max	24.0%	0.02%	2.40%	0.08%	0.02%	0.04%	3.20%

\*All welfare costs are expressed as a percentage of total lifetime consumption.

---

---

**Table 9. Welfare Loss with Higher Risk Aversion ( $\sigma = 5$ ):**

Income	Rep. Rate	Benefit Reform		Tax Reform		Double Uncertainty	
		saver	non-saver	saver	non-saver	saver	non-saver
		very low	67.5%	0.10%	1.83%	0.03%	0.04%
low	49.0%	0.06%	2.13%	0.04%	0.02%	0.06%	2.40%
average	36.4%	0.04%	2.22%	0.04%	0.01%	0.04%	2.60%
high	30.1%	0.03%	2.25%	0.04%	0.00%	0.03%	2.65%
max	24.0%	0.02%	2.26%	0.04%	0.00%	0.02%	2.68%

\*All welfare costs are expressed as a percentage of total lifetime consumption.

---

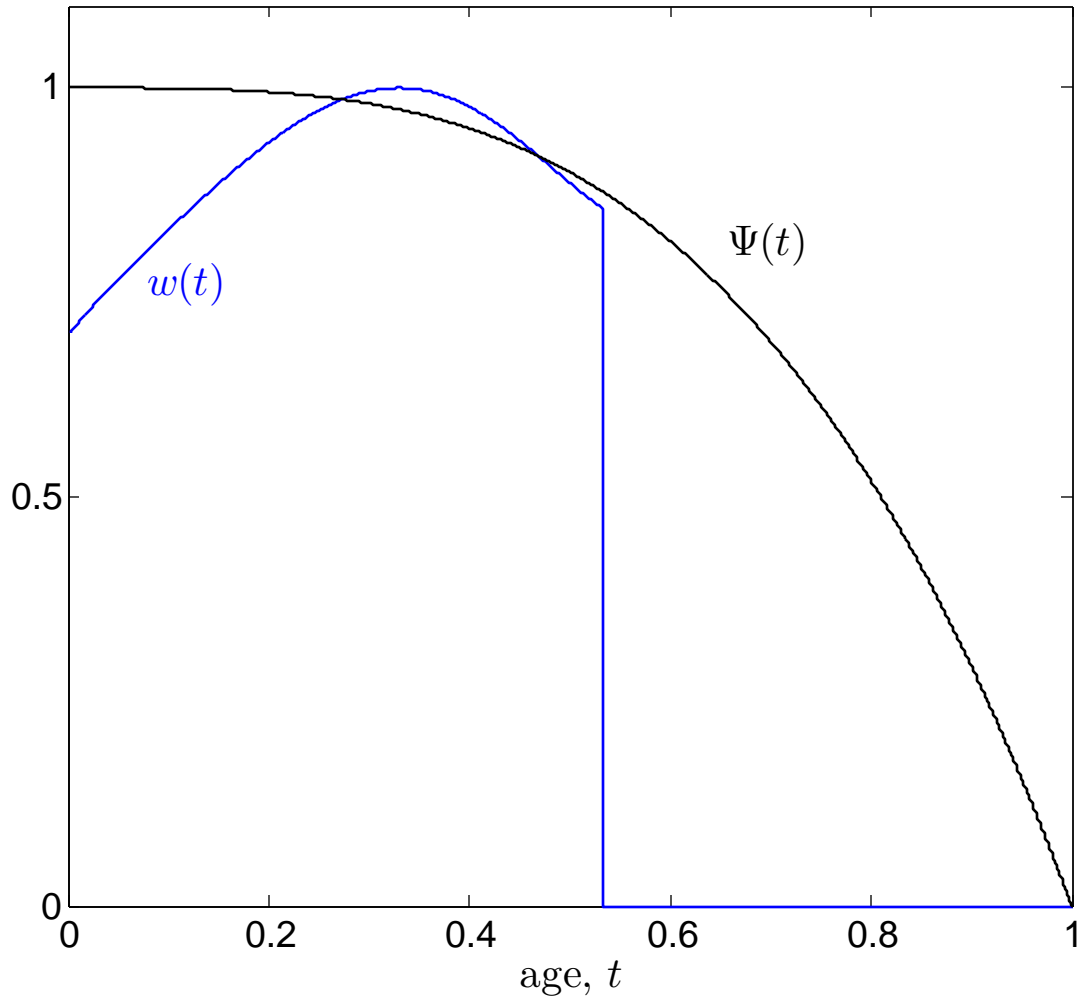
---

**Table 10. Welfare Loss with Higher Discounting** ( $\rho = r = 2.9\%$  per year):

Income	Rep.	Double					
	Rate	Benefit Reform		Tax Reform		Uncertainty	
		saver	non-saver	saver	non-saver	saver	non-saver
very low	67.5%	0.08%	0.47%	0.02%	0.04%	0.08%	0.43%
low	49.0%	0.05%	0.68%	0.02%	0.03%	0.04%	0.65%
average	36.4%	0.03%	0.89%	0.03%	0.02%	0.03%	0.87%
high	30.1%	0.02%	1.00%	0.03%	0.02%	0.02%	1.01%
max	24.0%	0.02%	1.11%	0.03%	0.02%	0.02%	1.16%

\*All welfare costs are expressed as a percentage of total lifetime consumption.

Figure 1. Baseline Wage Profile and Survival Uncertainty



Survival data: SSA 1990 cohort. Wage data: Gourinchas and Parker (2002).

Figure 2. Weibull Density Functions over Random Timing of Reform

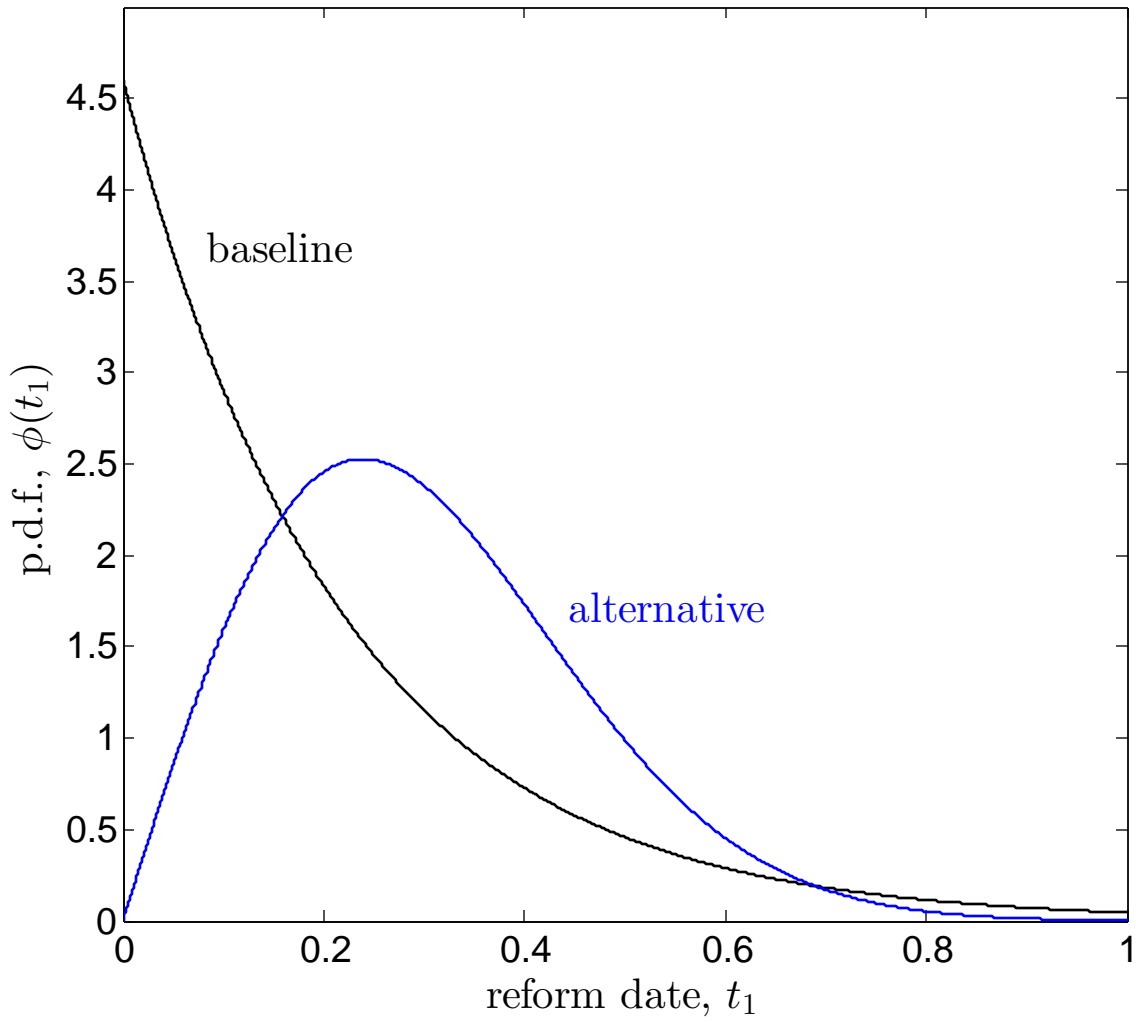
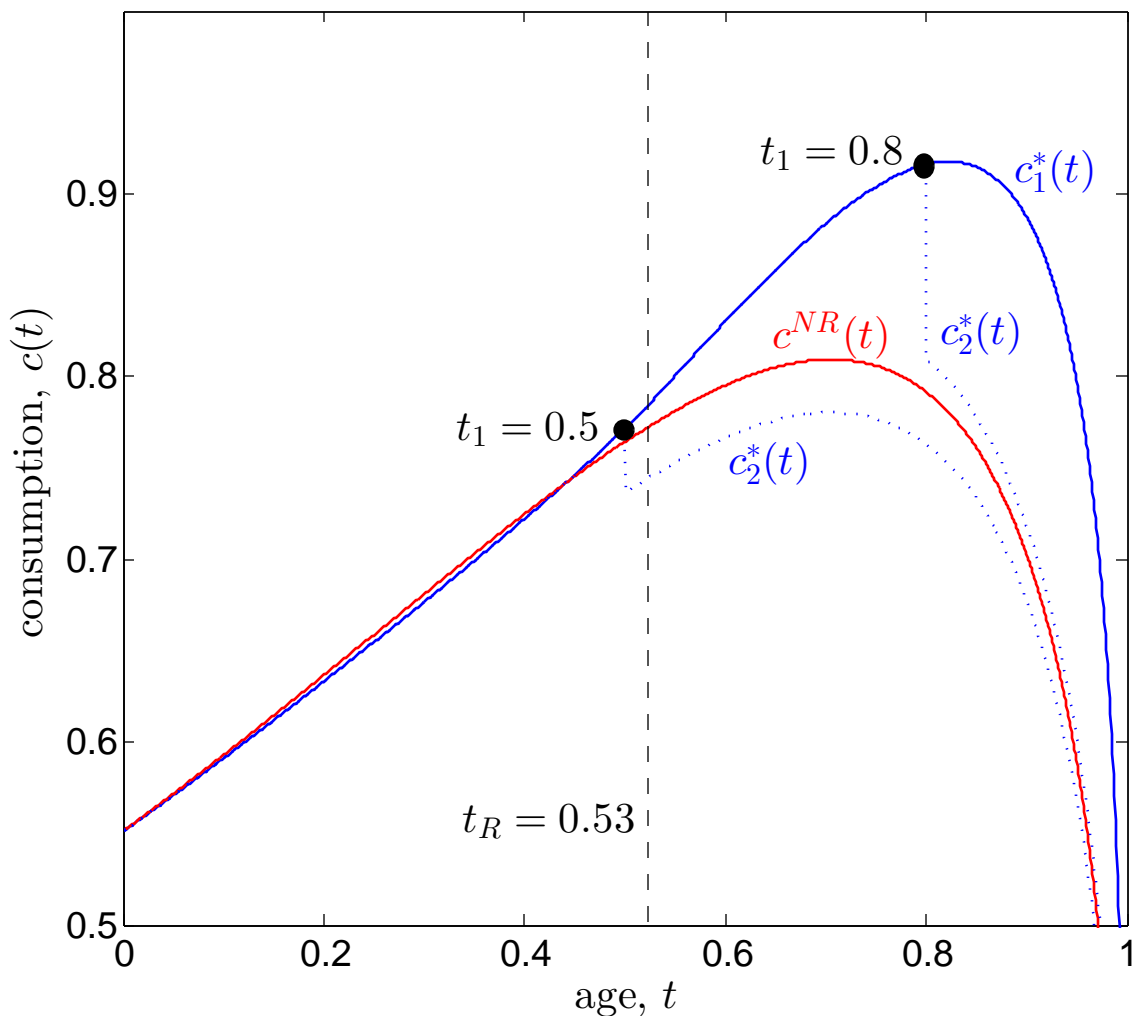


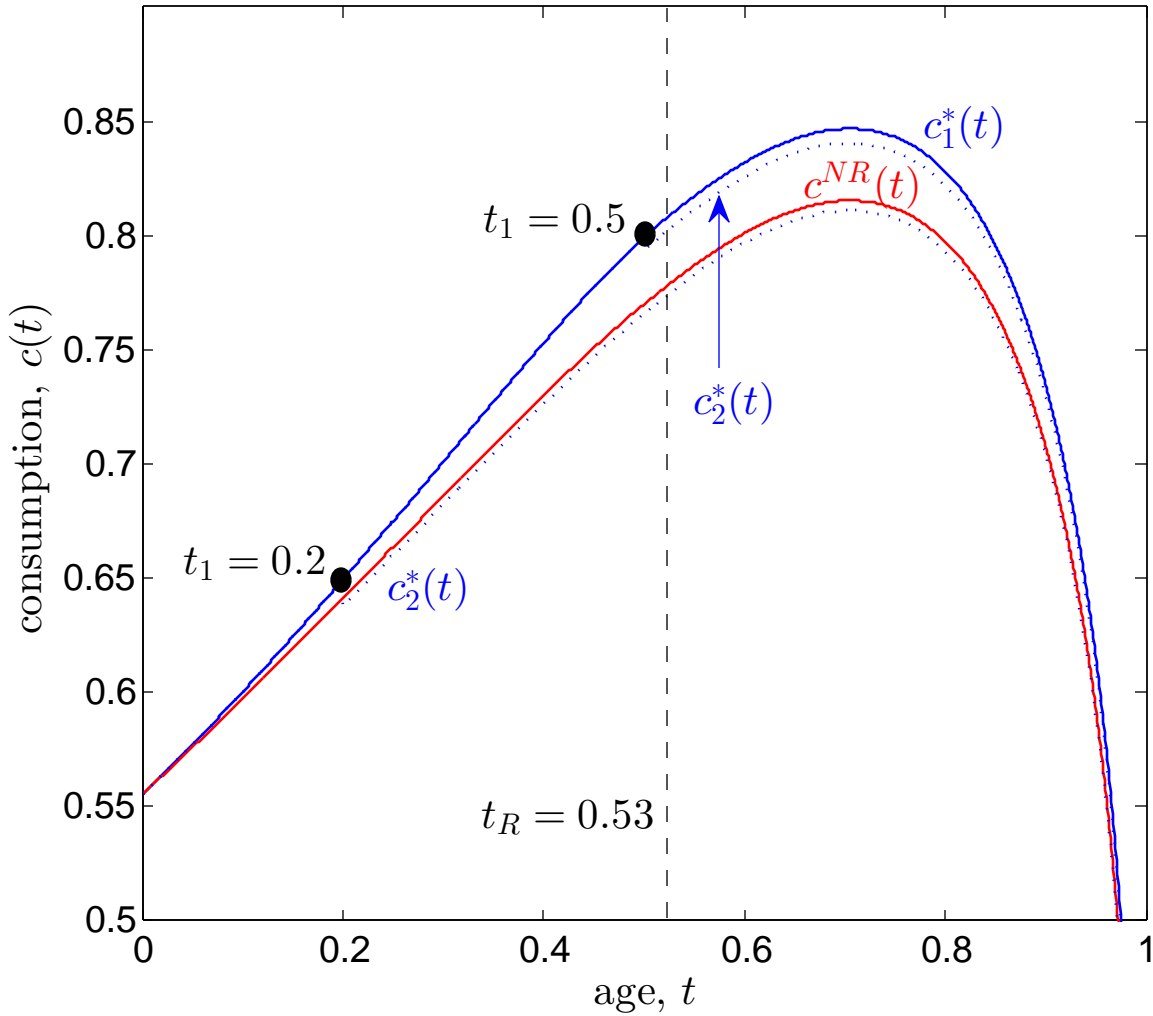


Figure 3. The Case of **Benefit Reform** with Stochastic Reform Date



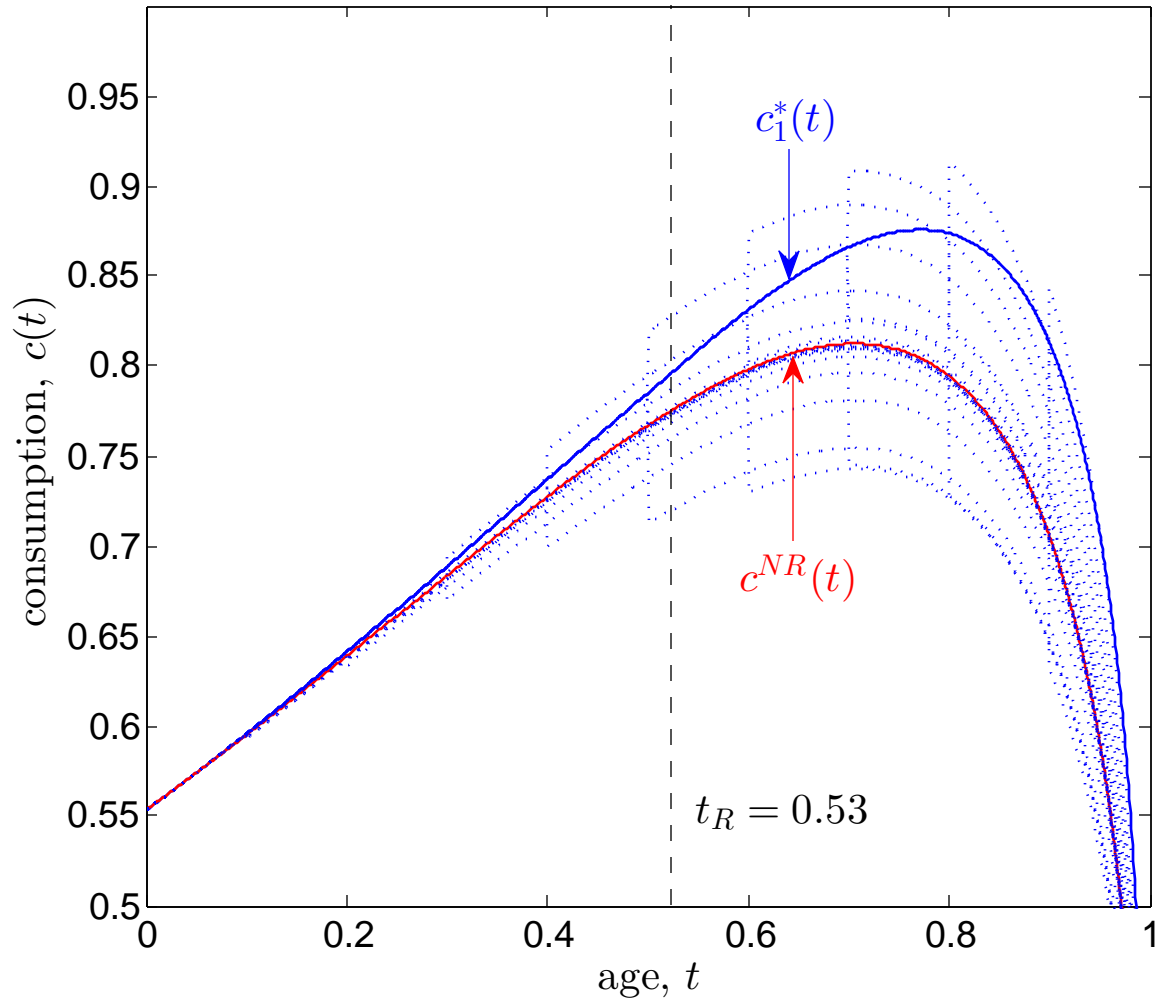
Note: While timing uncertainty is a continuous random variable, we show the optimal responses to just two potential shock dates  $t_1$ .

Figure 4. The Case of **Tax Reform** with Stochastic Reform Date



Note: While timing uncertainty is a continuous random variable, we show the optimal responses to just two potential shock dates  $t_1$ .

Figure 5. **Double Uncertainty: Timing and Structural Uncertainty**



Note: While double uncertainty is continuous, we show optimal responses to a few dates  $t_1 \in \{0.1, \dots, 0.9\}$  and structures  $\alpha \in \{0, 1/2, 1\}$ .