

1 Introduction

This appendix describes the components of the DSGE framework used in the paper. It is based on the medium-scale DSGE model of Smets and Wouters (2007) and Christiano, Eichenbaum and Evans (2005), with the segmented markets features of Andrés, López-Salido, Nelson (2004) and Chen, Cúrdia and Ferrero (2012). Data used in the estimation are also described here.

2 Model

2.1 Households

There is a continuum of investors of measure one. Two types of investors, $j = u, r$ consume C_t^j and dislike labor L_t^j . Consumption is relative to productivity Z_t . The utility function over the infinite horizon is given by

$$E_t \sum_{s=0}^{\infty} \beta_j^s b_{t+s}^j \left[\frac{1}{1 - \sigma_j} \left(\frac{C_{t+s}^j}{Z_{t+s}} - h \frac{C_{t+s-1}^j}{Z_{t+s-1}} \right)^{1 - \sigma_j} - \frac{\varphi_{t+s}^j (L_{t+s}^j(i))^{1 + \nu}}{1 + \nu} \right] \quad (1)$$

where β_j is the discount factor, b^j is the preference shock, σ_j is the CRRA, h is the habit parameter and φ^j is the labor supply shock of household j .

There are two types of bonds. The one-period bonds B_t pay a nominal return of R_t at time $t+1$. Long term bonds are perpetuities that cost $P_{L,t}$ at time t and pay an exponentially decaying coupon κ^s at time $t + s + 1$. Although the investors have identical preferences, a fraction ω_u of households trade in both short and long term government bonds. These are considered to be "unrestricted". They must however pay a transaction cost of ζ_t per unit of the bond purchased (it costs them $1 + \zeta_t$ dollars per unit, instead of 1 dollar). The remaining investors $1 - \omega_u = \omega_r$ only trade in long term bonds but pay no transaction costs (these are "restricted"). For the long-term bond, the price $P_{L,t} = 1 / (R_{L,t} - \kappa)$, where $R_{L,t}$ is the gross

yield on the bond. The bond duration is $R_{L,t}/(R_{L,t} - \kappa)$.

For the households, the flow budget constraint (FBC) is:

$$P_t C_t^u + B_t^u + (1 + \zeta_t) P_{L,t} B_{L,t}^u \leq R_{t-1} B_{t-1} + \sum_{s=1}^{\infty} \kappa^{s-1} B_{L,t-s}^u + W_t^u(i) L_t^u(i) + \mathcal{P}_t + \mathcal{P}_t^{cp} + \mathcal{P}_t^{fi} - T_t^u. \quad (2)$$

Here \mathcal{P}_t^{fi} are the per-capita nominal profits of the financial intermediary (who receives the transactions cost and distributes to the profits to all shareholders). \mathcal{P}_t and \mathcal{P}_t^{cp} are the profits from the ownership of intermediate goods produces and capital producers respectively. T_t^u are lump-sum taxes.

The FBC of the institutional investors is:

$$P_t C_t^r + P_{L,t} B_{L,t}^r \leq \sum_{s=1}^{\infty} \kappa^{s-1} B_{L,t-s}^r + W_t^r(i) L_t^r(i) + \mathcal{P}_t + \mathcal{P}_t^{cp} + \mathcal{P}_t^{fi} - T_t^r. \quad (3)$$

In the above expressions, the price of the final good is P_t , $W_t^j(i)$ is the wage of a household of type j . From the unrestricted households' optimization, FOC for short bonds is:

$$\Xi_t^u = \beta_u R_t E_t (\Xi_{t+1}^u), \quad (4)$$

and for the long bonds is:

$$\frac{1 + \zeta_t}{R_{L,t} - \kappa} \Xi_t^u = \beta_u E_t \left(\frac{R_{L,t+1}}{R_{L,t+1} - \kappa} \Xi_{t+1}^u \right). \quad (5)$$

Here Ξ_t^u is the Lagrange multiplier associated with the household's budget constraint.

The institutional investors only hold long bonds, and for these, the FOC is:

$$\frac{1}{R_{L,t} - \kappa} \Xi_t^r = \beta_r E_t \left(\frac{R_{L,t+1}}{R_{L,t+1} - \kappa} \Xi_{t+1}^r \right), \quad (6)$$

and Ξ_t^r is the Lagrange multiplier associated with the institutional investor's budget constraint.³

Both types of investors set their wages on a Calvo staggered basis (wages are reset with probability $1 - \zeta_w$), and this decision is based on the demand for their specific labor input

³Here the Lagrangian multiplier of investors of type i , $\Xi_t^{u,p}(i)$, is normalized to $\Xi_t^u = \Xi_t^{u,p}(i) Z_t P_t \forall i$.

$L_t(i)$. This is supplied to perfectly competitive labor agents, which aggregate the labor inputs into L_t :

$$L_t = \left[\int_0^1 L_i(i)^{\frac{1}{1+\lambda_w}} di \right]^{1+\lambda_w}, \quad (7)$$

where $\lambda_w > 0$ is the steady state wage markup. This implies the FOC for the demand of the labor input is:

$$L_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\frac{1+\lambda_w}{\lambda_w}} L_t \quad (8)$$

and the aggregate wage is

$$W_t = \left[\int_0^1 W_i(i)^{-\frac{1}{\lambda_w}} di \right]^{-\lambda_w}. \quad (9)$$

Then the labor supply decision of the household is:

$$\min_{\tilde{W}_t^u(i)} E_t \sum_{s=0}^{\infty} (\zeta_w \beta_u)^s b_{t+s}^u \frac{\varphi_{t+s}^u}{1 + \nu_u} L_{t+s}^u(i)^{1+\nu_u}$$

subject to the demand for labor, FBC in (2) and wage updating given by $W_{t+s}^u(i) = (\Pi e^\gamma)^s \tilde{W}_t^u(i)$. The first order conditions imply that the aggregate wage index can be rewritten as:

$$W_t = \left\{ (1 - \zeta_w) \left[\omega_u \left(\tilde{W}_t^u \right)^{\frac{-1}{\lambda_w}} + \omega_r \left(\tilde{W}_t^r \right)^{\frac{-1}{\lambda_w}} \right] + \zeta_w (\Pi e^\gamma W_{t-1})^{\frac{-1}{\lambda_w}} \right\}^{-\lambda_w}. \quad (10)$$

Wage setting: Households are monopolistic suppliers of differentiated labor inputs and set wages using Calvo. The fraction of households which do not re-set their wages index it to the steady state rate of inflation and productivity growth.

2.2 Final goods producers

The final good Y_t is made up of a composite of goods:

$$Y_t = \left[\int_0^1 Y_t(i)^{1/(1+\lambda_f)} di \right]^{(1+\lambda_f)}. \quad (11)$$

These final good producers buy intermediate goods, package them into Y_t , and sell the final good to consumers, investors and the government. Their maximization problem (in a perfectly competitive market) is:

$$\max_{Y_t, Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \quad (12)$$

subject to the constraint $Y_t = \left[\int_0^1 Y_t(i)^{1/(1+\lambda_f)} di \right]^{(1+\lambda_f)} \mu_{f,t}$. The FOCs imply:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{(1+\lambda_f)}{\lambda_f}} Y_t \quad (13)$$

and $P_t = \left[\int_0^1 P_t(i)^{-1/\lambda_f} di \right]^{-\lambda_f}$.⁴ Since the marginal cost is the same for all firms⁵, profits can be rewritten as $[P_t(i) - \lambda_{f,t} MC_t] Y_t(i)$, where $\lambda_{f,t}$ is the time-varying markup on prices, and evolves as:

$$\ln \lambda_{f,t} = \rho_{\lambda_f} \ln \lambda_{f,t-1} + \varepsilon_{\lambda,t}, \varepsilon_{\lambda,t} \sim N(0, \sigma_{\varepsilon_\lambda}^2). \quad (14)$$

With Calvo pricing, where each firm readjusts its prices with probability $1 - \zeta_p$ in each period, and those that do not, increase their prices $P_t(i)$ at the steady state rate of inflation π ⁶. The optimization problem for the firms that can adjust their prices is to choose a price level $\tilde{P}_t(i)$ to maximize expected discounted profits subject to the constraint $Y_{t+s}(i) = \left[\frac{\tilde{P}_t(i) \Pi^s}{P_{t+s}} \right]^{-\frac{1+\lambda_f}{\lambda_f}} Y_{t+s}$, where $\Pi = 1 + \pi$. From the resulting FOC and the fact that all firms that readjust their prices face the same optimization problem, the price⁷ is:

$$P_t = \left[(1 - \zeta_p) \tilde{P}_t^{-1/\lambda_f} + \zeta_p (\Pi P_{t-1})^{-1/\lambda_f} \right]^{-\lambda_f}. \quad (15)$$

⁴ λ_f is constrained to lie between $(0, \infty)$.

⁵ $MC_t = \left[W_t + R_t^k \frac{K_t(i)}{L_t(i)} \right] Z_t^{-(1-\alpha)} \left(\frac{K_t(i)}{L_t(i)} \right)^{-\alpha}$

⁶The rate of inflation is $\pi_t = P_t/P_{t-1}$.

⁷Here the i subscript has been dropped.

2.3 Intermediate goods producers

Intermediate good's producer i uses the technology:

$$Y_t(i) = Z_t^{1-\alpha} K_t(i)^\alpha L_t(i)^{1-\alpha}, \quad (16)$$

where $K_t(i)$ is the capital used in production and $L_t(i)$ is a composite labor input. Z_t is the total factor productivity, and $z_t = \log\left(\frac{Z_t/Z_{t-1}}{1+\gamma}\right)$ follows the process:

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t}, \varepsilon_z \sim N(0, \sigma_z^2). \quad (17)$$

The profit of the firm is given by $P_t(i) Y_t(i) - W_t L_t(i) - R_t^k K_t(i)$, where W_t is the aggregate nominal wage rate and R_t^k is the rental rate of capital. Combining the cost minimization FOCs yields:

$$K_t = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k} L_t. \quad (18)$$

2.4 Capital producers

All the households own a representative perfectly competitive firm which invests in capital, chooses utilization rate and rents the capital to intermediate firms. The utilization rate is u_t , and each period, an "effective" amount of capital is rented out:

$$K_t = u_t \bar{K}_{t-1}. \quad (19)$$

This utilization subtracts real resources, measured in terms of the consumption good: $a(u_t) \bar{K}_{t-1}$. Capital's law of motion is:

$$\bar{K}_t = (1 - \delta) \bar{K}_{t-1} + \mu_t \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) \right] I_t. \quad (20)$$

Here $\delta \in (0, 1)$ is the depreciation rate, S is the cost of adjusting investment. $S' > 0$ and $S'' > 0$. The capital producers maximize the expected discounted stream of dividends to their shareholders, subject to the constraint in 20.

2.5 Government

The flow budget constraint of the government is:

$$B_t + P_{L,t}B_{L,t} = R_{t-1}B_{t-1} + (1 + \kappa P_{L,t}) B_{L,t-1} + P_t G_t - T_t, \quad (21)$$

where the LHS is the nominal market value of the total debt issued by the government. The RHS is the cost of servicing bonds maturing at t along with government spending G_t and taxes T_t .

The composition of debt is controlled by the government as:

$$\frac{P_{L,t}B_{L,t}}{P_t Z_t} = S \left(\frac{P_{L,t-1}B_{L,t-1}}{P_{t-1}Z_{t-1}} \right)^{\rho_B} e^{\varepsilon_{B,t}}, \quad (22)$$

where $\varepsilon_{B,t}$ is an exogenous i.i.d. shock and $\rho_B \in (0, 1)$. S is a constant required to make the debt equation above an identity at steady state. The LSAP program will be interpreted as the shock to the composition of outstanding government liabilities relative to the past average.

The primary surplus of the government will evolve as:

$$\frac{T_t}{P_t Z_t} - \frac{G_t}{Z_t} \equiv \Phi_{z,t} = \Phi \left(\frac{P_{L,t-1}B_{LZ,t-1} + B_{Z,t-1}}{P_t B_{LZ,t} + B_{Z,t}} \right)^{\phi_T} e^{\varepsilon_{T,t}}, \quad (23)$$

where $\phi_T > 0$, $\varepsilon_{T,t}$ follows the stationary AR(1) process, $B_{Z,t} \equiv B_t/P_t Z_t$ and $B_{LZ,t} \equiv B_{L,t}/P_t Z_t$

2.6 Central Bank

In periods when the zero-lower bound is not binding, the Taylor rule (Taylor, 1993) with interest rate smoothing is followed:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_m} \left[\left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_t}{e^{4\gamma} Y_{t-1}} \right)^{\phi_y} \right]^{1-\rho_m} e^{\varepsilon_{m,t}}, \quad (24)$$

where $\varepsilon_{m,t}$ is an i.i.d. shock, and $\rho_m \in (0, 1)$, $\phi_\pi > 1$ and $\phi_y \geq 0$.

2.7 Intuition for the effect of LSAP

The baseline formulation of Cúrdia, Chen and Ferrero (2012) for the transaction cost is:

$$\zeta_t = \left(\frac{P_{L,t} B_{L,t}}{P_t Z_t} \right) \exp(\varepsilon_{\zeta,t}). \quad (25)$$

The authors use the Euler equations of the households and institutional investors to show:

$$E_t \left\{ \frac{\Xi_{t+1}^{p,u}}{\Xi_t^{p,u}} \left[\frac{P_{L,t+1}}{(1 + \zeta_t) P_{L,t}} R_{L,t+1} - \frac{P_{L,t+1}^{EH}}{P_{L,t}^{EH}} R_{L,t+1}^{EH} \right] \right\} = 0. \quad (26)$$

Here, $P_{L,t+1}^{EH}$ and $R_{L,t+1}^{EH}$ are the price and yield to maturity of the long-term bond in the absence of transactions costs, while holding the path for the marginal utility of consumption constant. Under the first order approximation, the risk premium is defined as the difference between the yields to maturity with and without transactions costs:

$$\widehat{RP}_t \equiv \widehat{R}_{L,t} - \widehat{R}_{L,t}^{EH}, \quad (27)$$

and combining the approximations of the no-arbitrage condition and the relation between the price and yields, the authors obtain:

$$\widehat{RP}_t = \frac{1}{D_L} \sum_{s=0}^{\infty} \left(\frac{D_L - 1}{D_L} \right)^s E_t \zeta_{t+s}. \quad (28)$$

2.8 Resource Constraints and Exogenous Processes

Budget constraint (BC) for the household is:

$$P_t C_t^u + B_t^u + (1 + \zeta_t) P_{L,t} B_{L,t}^u = R_{t-1} B_{t-1}^u + \frac{R_{L,t}}{R_{L,t} - \kappa} B_{L,t-1}^u + \int W_t^u(i) L_t^u(i) + \mathcal{P}_t + \mathcal{P}_t^{cp} + \mathcal{P}_t^{fi} - T_t^u, \quad (29)$$

and for the institutional investor is:

$$P_t C_t^r + P_{L,t} B_{L,t}^r \leq \frac{R_{L,t}}{R_{L,t} - \kappa} B_{L,t-1}^r + \int W_t^r(i) L_t^r(i) + \mathcal{P}_t + \mathcal{P}_t^{cp} + \mathcal{P}_t^{fi} - T_t^r. \quad (30)$$

The Government's BC is:

$$B_t + \frac{B_{L,t}}{R_{L,t} - \kappa} = R_{t-1}B_{t-1} + \frac{R_{L,t}}{R_{L,t} - \kappa}B_{L,t-1} + P_t G_t - R_t. \quad (31)$$

Since

$$\mathcal{P} = \int_t \mathcal{P}(i) di = \int P_t(i) Y_t(i) di - W_t L_t - R_t^K K_t, \quad (32)$$

it is the case that

$$\begin{aligned} W_t L_t &= \int W_t^u(i) L_t^u(i) di + \int W_t^r(i) L_t^r(i) di \\ P_t Y_t &= \int P_t(i) Y_t(i) di. \end{aligned} \quad (33)$$

The profits of the capital producers and financial institution are

$$\begin{aligned} R_t^K K_t - P_t a(u_t) \bar{K}_{t-1} - P_t I_t \\ \mathcal{P}^{fi} = \omega_u \frac{\zeta_t}{R_{L,t} - \kappa} B_{L,t}^u. \end{aligned} \quad (34)$$

The budget constraint is:

$$\omega_u C_t^u + \omega_r C_t^r + G_t + a(u_t) \bar{K}_{t-1} + I_t = Y_t. \quad (35)$$

The exogenous processes are specified as:

$$\begin{aligned} \text{Technology} &: z_t = \ln(e^{-\gamma} Z_t / Z_{t-1}) \\ z_t &= \rho_z z_{t-1} + \varepsilon_{z,t} \end{aligned}$$

$$\text{Preference for leisure: } \ln \varphi_t = \rho_\varphi \ln \varphi_{t-1} + \varepsilon_{\varphi,t}$$

$$\text{Price mark up: } \ln \lambda_{f,t} = \varepsilon_{\lambda,t}.$$

$$\text{Capital adjustment cost process: } \ln \mu_t = \rho_\mu \ln \mu_{t-1} + \varepsilon_{\mu,t}$$

$$\text{Intertemporal preference shifter: } \ln b_t = \rho_b \ln b_{t-1} + \varepsilon_{b,t}$$

Government spending: $\ln g_t = \rho_g \ln g_{t-1} + \varepsilon_{g,t}$

Exogenous risk premium shock: $\varepsilon_{\zeta,t} = \rho_{\zeta} \varepsilon_{\zeta,t-1} + \epsilon_{\zeta,t}$

The remaining shocks are the monetary policy shock ($\varepsilon_{m,t}$), fiscal shock ($\varepsilon_{T,t}$) and the long-term bond supply shock ($\varepsilon_{B,t}$).

3 Model Steady State

In steady state, the log of productivity grows at the rate γ and constant inflation is Π . Following Christiano et al. (2011), $a(u_t)$ is such that $u = 1$ in SS, and $a(1) = 0$. Also, $Y/Z = 1$, $\nu_u = \nu_r = \nu$, $S(e^\gamma) = S'(e^\gamma) = 0$. The ratio of the consumption and Lagrange multipliers associated with the maximization problems of the households and institutional investors are estimated. The levels of b^u and b^r are allowed to take whichever value required to allow the ratios to be consistent with each other and the resource constraints.

From the Euler equations:

$$1 = \beta_u R e^{-\gamma} \Pi^{-1}, \quad (36)$$

$$(1 + \zeta) = \frac{R_L}{R}, \quad (37)$$

$$\beta_u = \beta_r (1 + \zeta). \quad (38)$$

The level of long debt is determined by the risk premium:

$$B^{LMV}/Z = \zeta^{-1}(\zeta), \quad (39)$$

and taxes are determined by the government BC:

$$\frac{T}{Z} = \frac{G}{Z} - (1 - \beta_u^{-1}) \frac{B}{Z} - \left(\frac{1}{R_L - \kappa} - \frac{R_L}{R_L - \kappa} \frac{1}{e^\gamma \Pi} \right) \frac{B_L}{Z}. \quad (40)$$

The unit marginal efficiency of investment shock implies $q = 1$, and unit utilization implies $r^k = a'(1)$, which determines $a'(1)$ given r^k . The FOC for investment implies:

$$r^k = \bar{\beta}^{-1} e^\gamma - (1 - \delta). \quad (41)$$

Here

$$\bar{\beta} = \frac{\omega_u \beta_u \frac{\Xi^u}{\Xi^r} + \omega_r \beta_r}{\omega_u \frac{\Xi^u}{\Xi^r} + \omega_r}, \quad (42)$$

where Ξ^u/Ξ^r are derived from the Euler equations of the households (Ξ^u) and investors (Ξ^r).

Price setting implies that the marginal cost $mc = \frac{1}{1+\lambda_f}$, and the definition of marginal cost implies $w_z = \tilde{w}_z (r^k)^{\frac{\alpha}{1-\alpha}}$, where $\tilde{w}_z = (1 + \lambda_f)^{-\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} (1 - \alpha)$.

The technology function $L = \left(\frac{K}{Z}\right)^{\frac{\alpha}{1-\alpha}}$ and the capital demand implies that $L = \tilde{L} (r^k)^{\frac{\alpha}{1-\alpha}}$, where $\tilde{L} \equiv \left(\frac{\alpha}{1+\lambda_f}\right)^{\frac{\alpha}{1-\alpha}}$. The other relationships are:

$$\text{Effective capital: } \frac{\bar{K}}{Z} = e^\gamma \left(\frac{\alpha}{1 + \lambda_f}\right) (r^k)^{-1} \quad (43)$$

$$\text{Investment: } \frac{I}{Z} = [e^\gamma - (1 - \delta)] \left(\frac{\alpha}{1 + \lambda_f}\right) (r^k)^{-1} \quad (44)$$

$$\text{Resource constrain: } \omega_u \frac{C^u}{Z} + \omega_r \frac{C^r}{Z} = 1 - \frac{I}{Z} - \frac{G}{Z}. \quad (45)$$

The log-linear approximations of these relations are available upon request.

4 Data

In order to estimate the model, the relevant macroeconomic time series are constructed for January 1920 to December 1934. Balke and Gordon's (1986) Real GNP and GNP deflator series are used for the output and inflation measure. Population numbers are taken from the U.S. Census Bureau. The construction of the number of labor hours supplied entails two different data sources. From Beney's (1936) study, the series of average hours worked per week per worker in manufacturing is used to construct average actual hours per quarter per wage earner. This is multiplied with the average number of workers in manufacturing, available from the Bureau of Labor Statistics. Yields on bonds and notes are taken from the Banking and Monetary Statistics for 1914-1941 publication of the Federal Reserve, and the Federal Reserve's holdings of Treasury debt is constructed from the tables on Factors affecting bank reserves and condition statement of the Federal Reserve Banks. Following

Cúrdia, Chen and Ferrero (2012), the mapping of these variables to the states is:

$$\begin{aligned}
\Delta Y_t^{obs} &= 100 \left(\gamma + \hat{Y}_{z,t} - \hat{Y}_{z,t-1} + \hat{z}_t \right) \\
L_t^{obs} &= 100 \left(L + \hat{L}_t \right) \\
\Delta w_t^{obs} &= 100 \left(\gamma + \hat{w}_{z,t} - \hat{w}_{z,t-1} + \hat{z}_t \right) \\
\pi_t^{obs} &= 100 \left(\pi + \hat{\pi}_t \right) \\
r_t^{obs} &= 100 \left(r + \hat{r}_t \right) \\
r_{L,t}^{obs} &= 100 \left(r_L + \hat{r}_{L,t} \right) \\
B_t^{ratio,obs} &= \frac{P_L B_{L,z}}{B_z} \left(1 + \hat{P}_{L,t} + \hat{B}_{L,z,t} - \hat{B}_{z,t} \right)
\end{aligned}$$

where the state variables are in deviations from steady state values, $X_z = X/Z$ for variable X , and $\pi = \ln \Pi$, $r = \ln R$ and $r_L = \ln R_L$. The observable ΔY_t^{obs} corresponds to the first difference in logs of the real GDP series. Δw_t^{obs} is the first difference in logs of the real wage series, multiplied by 100.

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