

## Online Appendix to “The Shocks Matter: Improving our Estimates of Exchange Rate Pass-Through”

### Shock dependent exchange rate pass-through in an open economy model

In this appendix we develop a standard open-economy model in order to illustrate how and why pass-through depends on the underlying shocks moving the exchange rate. The model focuses exclusively on limited pass-through due to price rigidities and local-currency-pricing. While these are clearly not the only factors limiting pass-through, we focus on features that are most relevant for our empirical analysis of the UK over the past 20 years and for which there is strong evidence that they play an important role, as shown in Devereux and Yetman (2003) for the role of price stickiness and Gopinath (2015) for the role of the currency of invoicing.<sup>1</sup> We focus on understanding how pass-through depends on the underlying shocks moving the exchange rate, and, while pass-through could also vary with the degree of price stickiness or the currency of invoicing, we do not consider how variations in these affect pass-through here as these factors are likely to be less important in explaining variations within the UK over the period we consider. In Forbes *et al.* (2015), we show that our results are robust to a more complex model structure and that our main results are not sensitive to the specific parameter combination chosen for the illustration below.

#### 1. A standard open-economy model

Our model consists of a world composed of two countries, denoted  $H$  (Home) and  $F$  (Foreign). There are respectively  $n$  and  $1-n$  households in these countries. Households supply a homogenous labour input and consume both domestically-produced and imported goods. Firms produce differentiated goods using labour inputs and set prices in a staggered fashion. Some firms set the price of their exported goods in their own currency, while others set prices in foreign currency and are allowed to price discriminate between the domestic and export markets. The monetary authorities follow a persistent interest rate rule with a flexible domestic CPI inflation target. The model set-up is standard and close to that analysed in Benigno (2009) and in Corsetti *et al.* (2010). Readers familiar with those models can go directly to Section III.2 which explains why exchange rate pass-through may be incomplete in this model. In the following, we describe the equilibrium by focusing on domestic agents' behaviour, but foreign agents behave similarly.

##### 1.1. Households

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<sup>1</sup> Other factors which might be important in determining the extent and the pace of pass-through include the monetary policy regime and the flexibility of the exchange rate (Corsetti and Pesenti, 2004), the production structure (Corsetti *et al.*, 2010), the market structure (Devereux *et al.*, 2015), or the degree of habits in consumption (Jacob and Uuskula, 2016).

The representative domestic household aims to maximize its welfare ( $W$ ), which is a function of its expected future stream of discounted utility from private consumption ( $C$ ) and disutility from working ( $L$ ). The representative household's welfare is given by:

$$W_t = E_t \sum_{s=0}^{\infty} \beta^s \{U^c(C_{t+s}) - \kappa U^l(L_{t+s})\},$$

where  $E_t$  denotes the expectations at time  $t$ ,  $\beta$  is the discount factor, and  $\kappa$  is a parameter determining the weight put on labour vs. consumption fluctuations in affecting utility. The functional forms are as follows:

$$U^c(C_t) = \gamma_t^c \frac{C_t^{1-\sigma_H}}{1-\sigma_H}$$

$$U^l(L_t) = \frac{L_t^{1+\eta_H}}{1+\eta_H},$$

where  $\sigma_H > 0$  is the inverse of the inter-temporal elasticity of substitution and the relative risk aversion coefficient in the Home country, and  $\eta_H > 0$  is the inverse of the Frisch labour supply elasticity.  $\gamma_t^c$  is a demand (or preference) shock.

The household's consumption is a CES index of the composite good produced at Home for the Home market,  $C_H$ , and the composite good produced in the Foreign country for the Home market,  $C_F$ :

$$C_t = \left[ a_H^{\frac{1}{\varphi_H}} C_{H,t}^{\frac{\varphi_H-1}{\varphi_H}} + (1-a_H)^{\frac{1}{\varphi_H}} C_{F,t}^{\frac{\varphi_H-1}{\varphi_H}} \right]^{\frac{\varphi_H}{\varphi_H-1}}, \quad 0 < a_H < 1, \varphi_H > 0, \quad (1)$$

where the constant elasticity of substitution between the Home and Foreign goods is denoted  $\varphi_H$ .  $a_H$  is the weight given to consumption of the composite Home good and is defined as  $a_H \equiv 1 - (1-n)op$  where  $op$  is a measure of openness. Similarly,  $1 - a_H \equiv (1-n)op$  is the weight attached to consumption of the composite Foreign good. If  $op < 1$  so that  $a_H > n$ , then a home bias in consumption is present.

The composite domestic and Foreign goods, destined for the Home market, are assumed to be composed of differentiated goods denoted  $C_{H,t}(h)$  and  $C_{F,t}(f)$ , which are imperfectly substitutable with constant elasticity of substitution  $\theta_H$  and  $\theta_F$ :

$$C_{H,t} = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\theta_H}} \int_0^n C_{H,t}(h)^{(\theta_H-1)/\theta_H} dh \right]^{\theta_H/(\theta_H-1)},$$

$$C_{F,t} = \left[ \left( \frac{1}{1-n} \right)^{1/\theta_F} \int_n^1 C_{F,t}(f)^{(\theta_F-1)/\theta_F} df \right]^{\theta_F/(\theta_F-1)}.$$

Households choose their relative traded consumption demand such as to maximize utility for given expenditures. The resulting domestic demand for, respectively, Home and Foreign traded goods are:

$$C_{H,t} = a_H \left( \frac{P_{H,t}}{P_t} \right)^{-\varphi_H} C_t, \quad (2)$$

$$C_{F,t} = (1 - a_H) \left( \frac{P_{F,t}}{P_t} \right)^{-\varphi_H} C_t, \quad (3)$$

where  $P_H$  and  $P_F$  respectively denote the price of the domestically -produced generic good  $C_H$  and the foreign good  $C_F$  in domestic currency, whereas  $P$  denotes the price of the domestic consumption basket  $C$ .

The consumption-based price indices are defined analogously to the consumption bundles

$$P_t = [a_H P_{H,t}^{1-\varphi_H} + (1 - a_H) P_{F,t}^{1-\varphi_H}]^{\frac{1}{1-\varphi_H}}, \quad (4)$$

where

$$P_{H,t} = \left[ \frac{1}{n} \int_0^n p_t(h)^{1-\theta_H} dh \right]^{\frac{1}{1-\theta_H}}, \quad P_{F,t} = \left[ \frac{1}{1-n} \int_0^{1-n} p_t(f)^{1-\theta_F} df \right]^{\frac{1}{1-\theta_F}}.$$

Domestic households face complete financial markets at the domestic level; they own an equal share in every domestic firm and profits are therefore equally distributed among households. Households also have access to the international financial markets, but these are incomplete; only nominal one-period bonds denominated in Foreign currency are traded across countries. The interest on these internationally traded bonds depends on the Foreign interest rate and the level of external debt: the yields of the bonds are increasing in external debt, as in Schmitt-Grohe and Uribe (2003). Apart from implying stationarity of the steady state, modelling financial frictions through a debt-elastic yield on bonds allows for state-contingent yield differences across countries.

Every period, households use their labour income, wealth accumulated in domestic and foreign bonds (denominated in Foreign currency), and profits of firms in the domestic economy ( $PR$ ) to purchase consumption and both domestically-issued bonds ( $B_H$ ) and Foreign bonds ( $B_F$ ) and pay lump-sum taxes. The representative household budget constraint thus amounts to:

$$C_t + \frac{B_{H,t}}{P_t(1+i_t)} + \frac{s_t B_{F,t}}{P_t(1+i_t^*) \Phi \left( \frac{s_t B_{F,t}}{P_t} \right)} + T_t = \frac{W_t}{P_t} L_t + \frac{B_{H,t-1}}{P_t} + \frac{s_t B_{F,t-1}}{P_t} + PR_t, \quad (5)$$

where  $i_t$  is the nominal interest set by the Home central bank in period  $t$  and defines the return on domestically-issued bonds denominated in the Home currency ( $B_H$ ), and  $i_t^*$  is the nominal interest set by the Foreign central bank in period  $t$  (starred variables denote Foreign variables),  $s_t$  is the nominal exchange rate,  $W_t$  is the nominal wage rate,  $L_t$  is hours worked,  $PR_t$  denotes profits made by domestic firms,  $T_t$  denotes lump-sum taxes paid by the household, and  $B_{F,t}$  is the nominal holdings of

Foreign bonds (denominated in Foreign currency). The function  $\Phi$  is assumed to depend positively on deviations of external debt from its steady state level,  $\Phi'(\cdot) < 0$ , and satisfies  $\Phi\left(\frac{sB_F}{P}\right) = 1$  in steady state.<sup>2</sup>

The first-order conditions of the representative domestic household's maximisation problem with respect to consumption, bond holdings and labour supply can be aggregated to yield:

$$\beta E_t \frac{\gamma_{t+1}^C C_{t+1}^{-\sigma} (1+i_t)}{\gamma_t^C C_t^{-\sigma} \pi_{t+1}} = 1 \quad (6)$$

$$\beta E_t \frac{\gamma_{t+1}^C C_{t+1}^{-\sigma} (1+i_t^*) Q_{t+1}}{\gamma_t^C C_t^{-\sigma} \pi_{t+1}^* Q_t} = \frac{1}{\Phi\left(\frac{s_t B_{F,t}}{P_t}\right)} \gamma_t^S \quad (7)$$

$$\frac{L_t^\eta}{\gamma_t^C C_t^{-\sigma}} = \frac{W_t}{P_t} \quad (8)$$

where  $\pi_t \equiv \frac{P_t}{P_{t-1}}$  and  $\pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$ , with  $P_t^*$  the Foreign consumer price index, denote CPI inflation respectively in the Home and in the Foreign country, and  $Q_t \equiv \frac{s_t P_t^*}{P_t}$  is the real exchange rate.<sup>3</sup>  $\gamma_t^S$  represents an exogenous shock to the nominal exchange rate. The shock pushes the equilibrium away from uncovered interest parity.

## 1.2. Firms

Firms produce differentiated goods using a technology which is linear in labour, so that output of domestic firm  $h$  is  $y_t(h) = l_t(h)$ . Firms are monopolistically competitive and set prices in a staggered fashion à la Calvo-Yun. That is, firms reset their price at a time-independent random frequency. More specifically, each firm faces the probability  $1-\alpha_H^k$  of being able to reset its price in each period.

A proportion  $\gamma_H^{PCP}$  of firms set the price of their exports in their own currency and do not discriminate across markets (i.e., engages in producer-currency pricing, PCP), while the remainder of firms set their export prices in the currency of the destination market and may set different prices in the two markets (i.e., engages in local-currency pricing, LCP). The price index of domestic goods is

<sup>2</sup> We specify the yield premium associated with holding bonds to be linear in deviations of borrowing/lending from steady state:  $\Phi(b_t) = 1 - \delta(b_t - \frac{B}{P})$ , with  $\delta > 0$  and  $\frac{B}{P} = \frac{sB_F}{P}$ . Note that because  $\Phi'(\cdot) < 0$ , whenever  $B_F$  is low, then the yield on debt is high ( $\Phi(\cdot) > 1$ ). On the contrary, when bond holdings are high implying that Home households have claims on Foreign households, then  $\Phi(\cdot) < 1$  and the price of bonds is high and purchasing even more bonds is expensive. For simplicity, we assume that individual households do not internalize the effect of changes in their own bond holdings on the yield, i.e. they take the function  $\Phi(\cdot)$  as given.

<sup>3</sup> Note that the Foreign household only faces one Euler equation as it holds only its own internationally traded bonds. This assumption can be justified by the fact that most small open economies have the majority of their international debt denominated in the currency of a larger economy. Allowing for international trade in a second bond denominated in the Home currency would not change the results.

therefore  $P_{H,t} \equiv \left[ \gamma_H^{PCP} P_{H,t}^{PCP^{1-\theta_H}} + (1 - \gamma_H^{PCP}) P_{H,t}^{LCP^{1-\theta_H}} \right]^{\frac{1}{1-\theta_H}}$ , where  $P_{H,t}^{PCP}$  ( $P_{H,t}^{LCP}$ ) is the price of goods produced by PCP (LCP) firms, and the price index of imports is  $P_{F,t} \equiv \left[ \gamma_F^{PCP} S_t P_{F,t}^{PCP^*^{1-\theta_F}} + (1 - \gamma_F^{PCP}) P_{F,t}^{LCP^*^{1-\theta_F}} \right]^{\frac{1}{1-\theta_F}}$ .

The optimisation problem of the PCP firm producing good  $h$  and getting the opportunity to reset its price at time  $t$  consists in choosing a price  $p_t^{PCP}(h)$  such as to maximize expected discounted future profits:

$$\max_{p_t^{PCP}(h)} E_t \sum_{s=0}^{\infty} \alpha_H^s \mu_{t,t+s} \left[ \left( (1 - \tau_H) p_t^{PCP}(h) - W_{t+s} \right) y_{t,t+s}^{PCP}(h) \right],$$

where  $E_t$  is the expectations operator,  $\mu_{t,t+s}$  is the stochastic discount factor of the firm, and  $\tau_H$  is a tax on sales.  $y_{t,t+s}^{PCP}(h)$  is the domestic and foreign demand at time  $t + s$  for good  $h$  at the price  $p_t^{PCP}(h)$ :  $y_{t,t+s}^{PCP}(h) = \left( \frac{p_t^{PCP}(h)}{P_{H,t+s}} \right)^{-\theta_H} C_{H,t+s} + \frac{1-n}{n} \left( \frac{p_t^{PCP}(h)}{S_{t+s} P_{H,t+s}^*} \right)^{-\theta_H} C_{H,t+s}^*$  where  $P_{H,t}^* \equiv \gamma_H^{PCP} \frac{P_{H,t}^{PCP}}{S_t} + (1 - \gamma_H^{PCP}) P_{H,t}^{LCP^*}$  is the import price index in the Foreign economy. Given that firms are owned by the households, their stochastic discount factor is identical to the stochastic discount factor of the representative household:  $\mu_{t,t+s} = \beta^s \left( \frac{U_{C,t+s}}{P_{t+s}} \right) / \left( \frac{U_{C,t}}{P_t} \right)$ , where  $\beta$  is the households' discount factor and  $U_{C,t}$  is the households' marginal utility from consumption in period  $t$ .

The resulting first order conditions imply that prices are set according to expectations of future marginal costs and demand in the following way:

$$p_t^{PCP}(h) = \frac{\theta_H}{(\theta_H - 1)(1 - \tau_H)} \frac{E_t \sum_{s=0}^{\infty} (\beta \alpha_H)^s P_{t+s}^{-1} U_{C,t+s} W_{t+s} y_{t,t+s}^{PCP}(h)}{E_t \sum_{s=0}^{\infty} (\beta \alpha_H)^s P_{t+s}^{-1} U_{C,t+s} y_{t,t+s}^{PCP}(h)}. \quad (9)$$

Because all PCP firms that reset their price in a given period face the same expectations of marginal costs and demand, they all set the same price. Hence, the price index of PCP firms,  $P_H^{PCP}$ , is given by:

$$P_{H,t}^{PCP} = \left[ \alpha_H^k P_{H,t-1}^{PCP^{1-\theta_H}} + (1 - \alpha_H^k) p_t^{PCP}(h)^{1-\theta_H} \right]^{\frac{1}{1-\theta_H}}. \quad (10)$$

LCP firms can set different prices across the domestic and foreign markets and thus face two optimisation problems. The choice of the domestic price is chosen by solving the following maximisation problem:

$$\max_{p_t^{LCP}(h)} E_t \sum_{s=0}^{\infty} \alpha_H^s \mu_{t,t+s} \left[ \left( (1 - \tau_H) p_t^{LCP}(h) - W_{t+s} \right) y_{t,t+s}^{LCP}(h) \right],$$

where  $y_{t,t+s}^{LCP}(h)$  is the domestic demand at time  $t + s$  for good  $h$  at the price  $p_t^{LCP}(h)$ :

$y_{t,t+s}^{LCP}(h) = \left( \frac{p_t^{LCP}(h)}{P_{H,t+s}} \right)^{-\theta_H} C_{H,t+s}$ . The first-order condition can be written as:

$$\frac{p_t^{LCP}(h)}{P_{H,t}^{LCP}} = \frac{\theta_H}{(\theta_H - 1)(1 - \tau_H)} \frac{E_t \sum_{s=0}^{\infty} (\beta \alpha_H)^s P_{t+s}^{-1} U_{C,t+s} W_{t+s} y_{t,t+s}^{LCP}(h)}{E_t \sum_{s=0}^{\infty} (\beta \alpha_H)^s P_{H,t}^{LCP} P_{t+s}^{-1} U_{C,t+s} y_{t,t+s}^{LCP}(h)}. \quad (11)$$

Finally, the choice of the export price is chosen by solving the following maximisation problem:

$$\max_{p_t^{LCP^*}(h)} E_t \sum_{s=0}^{\infty} \alpha_H^s \mu_{t,t+s} \left[ \left( (1 - \tau_H) s_{t+s} p_t^{LCP^*}(h) - W_{t+s} \right) y_{t,t+s}^{LCP^*}(h) \right],$$

where  $y_{t,t+s}^{LCP^*}(h)$  is the foreign demand at time  $t + s$  for good  $h$  at the foreign currency price  $p_t^{LCP^*}(h)$ :

$y_{t,t+s}^{LCP^*}(h) = \left( \frac{p_t^{LCP^*}(h)}{P_{H,t+s}^*} \right)^{-\theta_H} C_{H,t+s}^*$ . The resulting first-order condition is:

$$\frac{p_t^{LCP^*}(h)}{P_{H,t}^{LCP^*}} = \frac{\theta_H}{(\theta_H - 1)(1 - \tau_H)} \frac{E_t \sum_{s=0}^{\infty} (\beta \alpha_H)^s P_{t+s}^{-1} U_{C,t+s} W_{t+s} y_{t,t+s}^{LCP^*}(h)}{E_t \sum_{s=0}^{\infty} (\beta \alpha_H)^s s_{t+s} P_{H,t}^{LCP^*} P_{t+s}^{-1} U_{C,t+s} y_{t,t+s}^{LCP^*}(h)}. \quad (12)$$

Aggregating output across firms yields  $Disp_t Y_t = L_t$  where  $Disp_{k,t} \equiv \int_0^n \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\theta_H^k} dh \geq 1$  is a measure of the degree of price dispersion.

### 1.3. Fiscal and monetary authorities

The government levies taxes and re-distributes them to households as lump sum transfers so that it balances its budget every period.

$$-T_t = \tau_H P_{H,t} Y_{H,t}. \quad (13)$$

Sales taxes are fixed to ensure that the steady state is efficient:

$$\tau_H = \frac{1}{1 - \theta_H}. \quad (14)$$

We abstract from monetary frictions and can thus consider a "cashless economy", as in Woodford (2003). The domestic monetary policy instrument is the nominal interest rate paid on one-period bonds, denoted  $i$ . The monetary policy authority sets the interest rate on domestic bonds with an aim to stabilize domestic CPI inflation and smooth interest rate changes. In particular, the Home monetary authority follows a rule of the following form:

$$\log \left( \frac{1+i_t}{\bar{i}} \right) = (1 - \alpha_H^\pi) \log \left( \frac{1+i_{t-1}}{\bar{i}} \right) + \alpha_H^\pi \log \left( \frac{\pi_t}{\bar{\pi}} \right) + \gamma_t^I, \quad (15)$$

where  $\alpha_H^\pi$  indicates the relative weight put on inflation targeting.  $\psi_t^I$  is a monetary policy shock. The Foreign monetary authority follows an analogous monetary policy rule. Monetary policy affects the real economy in the presence of nominal rigidities and through its effect on countries' debt burdens.

### 1.4. Goods and asset market equilibrium

Aggregate demand facing domestic producers of traded goods amounts to:

$$Y_{H,t} = a_H \left( \frac{P_{H,t}}{P_t} \right)^{-\varphi} C_t + \frac{1-n}{n} (1 - a_F) \left( \frac{P_{H,t}}{P_t} \right)^{-\varphi} Q_t^\varphi C_t^*, \quad (16)$$

and aggregate demand for foreign traded goods amounts to:

$$Y_{F,t} = \frac{n}{1-n} (1 - a_H) \left( \frac{P_{F,t}^*}{P_t^*} \right)^{-\varphi} Q_t^{-\varphi} C_t + a_F \left( \frac{P_{F,t}^*}{P_t^*} \right)^{-\varphi} C_t^*. \quad (17)$$

Output is demand-determined in equilibrium, and, hence, the above equations can also be viewed as goods market clearing conditions.

Equilibrium in the financial markets requires that bonds and assets issued in the Home economy are in zero net supply within the domestic economy,

$$B_{H,t} = 0, \quad (18)$$

and that internationally traded bonds issued in Foreign currency by the Foreign country are in zero net supply:

$$nB_{F,t} + (1 - n)s_t B_{F,t}^* = 0, \quad (19)$$

where  $B_{F,t}^*$  denotes Foreign holdings of the Foreign bond.

Appendix A presents the equilibrium equations for this standard open-economy model.

## 2. Exchange rate pass-through in the model

In this model, pass-through to import prices will not be full (i.e., 100%) for several reasons. First, some foreign exporters set their price in the Home currency (i.e., they are local-currency pricing or LCP exporters), but face sticky prices and are therefore not able to change their price immediately after a change in the exchange rate. The import price of goods produced by these foreign LCP firms will therefore only adjust sluggishly to changes in the exchange rate. This is true whatever shock hits, and pass-through to import prices is therefore always going to be less than 100 percent in the short run in the presence of LCP exporters.

But there are other reasons why pass-through to import prices may not be full – even when exporters eventually adjust their prices. Exporters set their prices in a forward-looking manner to reflect their expected marginal costs and expected demand conditions. If these marginal costs and demand conditions are expected to change as a result of the shock, exporters might choose to reflect that in their prices and adjust their mark-ups instead. Because these determinants of exporters' pricing decisions will be affected differently by different shocks, the extent to which exchange rate changes get reflected into prices will depend on the shock. It will also depend on the monetary policy

response and the persistence of the exchange rate movement. Therefore, within this relatively standard framework, the degree to which exporters pass through any move in the exchange rate to the import price – or instead adjust their mark-ups – depends on the shock which caused its move.

Within our model, the import price level in period  $t$  ( $P_{F,t}$ ) is a function of: the exchange rate ( $s_t$ ); marginal costs faced by foreign exporters, which depend on foreign wages ( $W_t^*$ ); and the mark-up over marginal costs ( $mkup_t^*$ ). In other words,

$$P_{F,t} = s_t mkup_t^* W_t^* .$$

We can then decompose any change in the import price level (relative to the level of prices) into changes in the real exchange rate, changes in marginal costs, and changes in the average mark-up over marginal costs:

$$\frac{\widehat{P_{F,t}}}{\widehat{P_t}} = \widehat{Q_t} + \widehat{mkup_t^*} + \frac{\widehat{W_t^*}}{\widehat{P_t^*}} , \quad (20)$$

where hatted variables denote deviations from steady state. If exchange rate pass-through was full, then the mark-up charged by exporters would not change when the exchange rate changed. Instead, the import price level would adjust to the change in the exchange rate and the potential change in foreign marginal costs.

### 3. A quantitative illustration of shock-dependent exchange rate pass-through

Next, we illustrate how pass-through might differ according to the drivers of the fluctuations in the exchange rate by considering the impact of three domestic shocks in this model: a shock to demand (a preference shock), a monetary policy shock, and a shock to the UIP condition (an exogenous exchange rate shock). We assume that the shocks follow first order autoregressive processes with i.i.d. normal innovations. In our model simulations of these shocks, we restrict the Home country to be a small open economy producing 5% of world GDP in steady state. The other parameter values of the model are listed in Table 2 and are fairly standard in the literature, see Corsetti *et al.* (2010). The degree of openness is chosen to be 30%, in line with the UK's import share of CPI.<sup>4</sup> We assume that 60% of foreign exporters set prices in the local currency and may discriminate between markets, while the rest set prices in their own currency. The Calvo price stickiness parameter

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<sup>4</sup> This 30% imported share of the CPI is calculated by the UK's Office of National Statistics (ONS) and should include both imported goods as well as the share of imported content in non-traded goods (to the best that it can be calculated).



is chosen to be 0.6, which implies that 40% of firms get the opportunity to reset their price every quarter.

There is obviously uncertainty about the precise estimates of the structural parameters in our model. Forbes *et al.* (2015) takes this into account and shows that our results do not hinge on a particular parameter combination. We have also checked that the persistence of shocks, while potentially important in affecting exchange rate pass-through, does not affect our main results.

**Table 2: Parameter values**

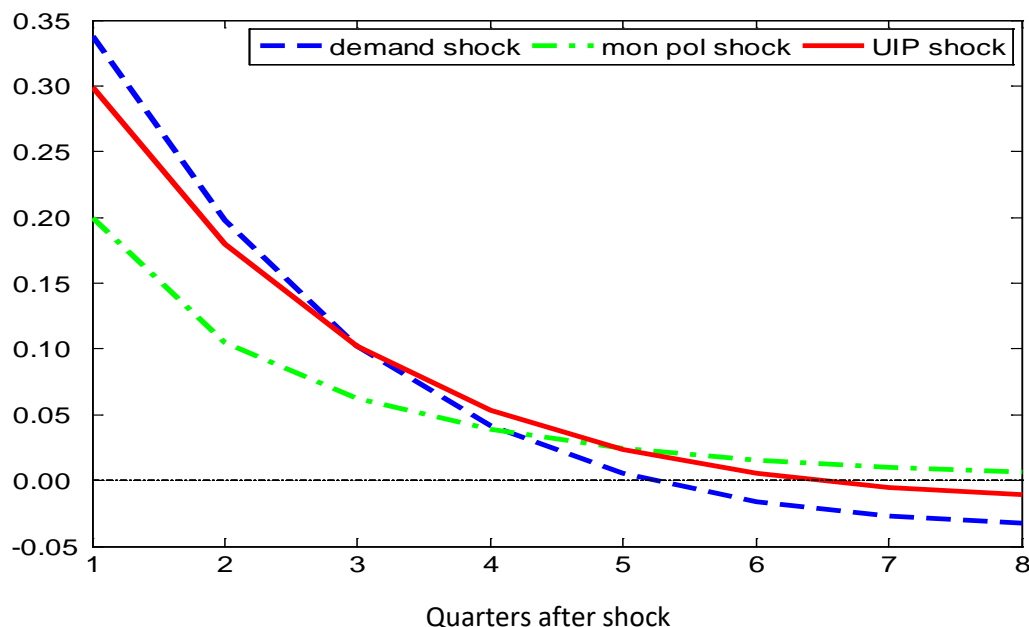
<b>Description</b>	<b>Parameter</b>	<b>Value</b>
Population in Home country	$n$	0.05
Discount factor	$\beta$	0.99
Yield sensitivity to external debt	$\delta$	0.01
Degree of openness	$op_H, op_F$	0.3
Inverse of the Frish elasticity of labour supply	$\eta_H, \eta_F$	2
Risk aversion coefficient	$\sigma_H, \sigma_F$	1.1
Price stickiness parameter	$\alpha_H, \alpha_F$	0.6
Intra-temporal elasticity of substitution	$\theta_H, \theta_F$	7
Elasticity of substitution between H and F goods	$\phi_H, \phi_F$	0.75
Proportion of firms setting export prices in their own currency (PCP)	$\gamma_H, \gamma_F$	0.4
<b><u>Home/Foreign monetary policy rule parameters:</u></b>		
Interest rate persistence	$\alpha_H^R, \alpha_F^R$	0.7
Interest rate sensitivity to CPI inflation	$\alpha_H^\pi, \alpha_F^\pi$	1.5
<b><u>Shock processes</u></b>		
Persistence parameter for demand shocks		0.9
Persistence parameter for UIP shocks		0.9

Using the decomposition in equation (20), we investigate how import prices respond to changes in the exchange rate caused by different shocks. First, consider how import prices change following a shock which only changes the exchange rate but not any other fundamentals – an *exogenous exchange rate shock* (or UIP shock). This shock affects neither exporters' marginal costs nor the demand conditions they face directly and can therefore serve as a benchmark.<sup>5</sup> As already noted, those foreign exporters which are able to change prices will do so. Only a certain proportion of exporters get the opportunity to change their price in a given quarter, however, and therefore the adjustment of average import prices to the exchange rate will be sluggish. This implies that the average exporting firm does not fully pass through the exchange rate movement into import prices,

<sup>5</sup> The exchange rate change does not affect foreign marginal costs, as the foreign economy is assumed to be very large compared to the domestic economy.

but instead adjusts its mark-up. The solid red line in Figure 2 shows the estimated changes to mark-ups over eight quarters after such an exogenous exchange rate shock that causes a 1% appreciation.

**Figure 1: Foreign exporters' mark-up after selected shocks**



**Note:** The figure depicts the percentage change in foreign exporters' average mark-up following a shock which appreciates the exchange rate by 1%.

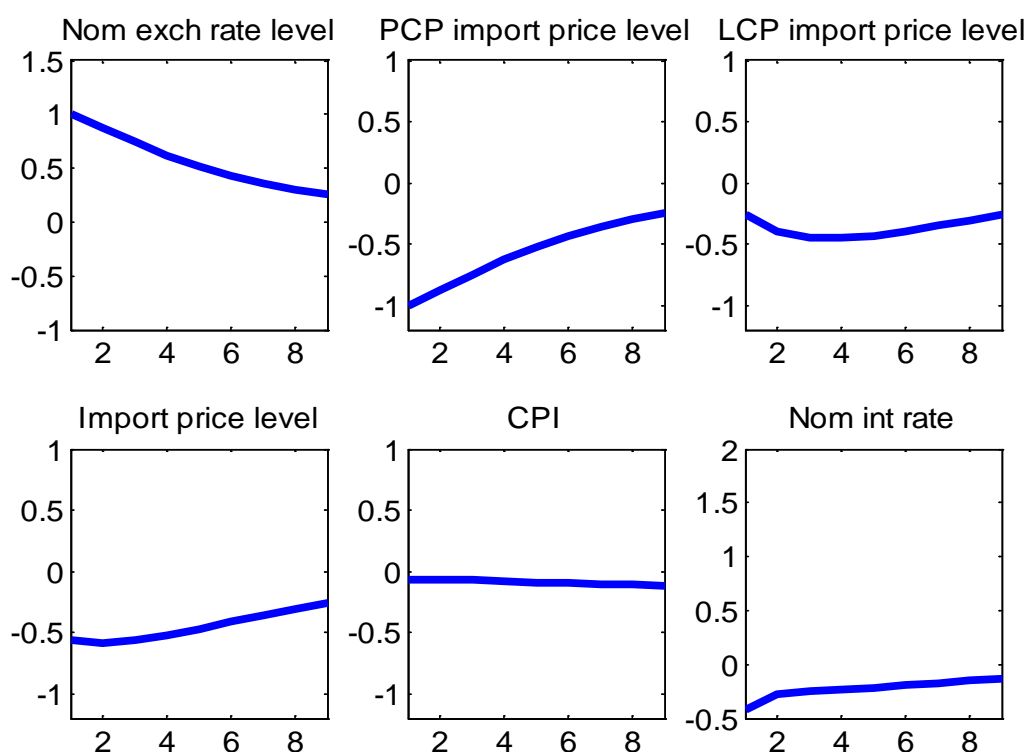
Figure 2 provides more details on the adjustments that occur in response to this 1% appreciation caused by the exogenous UIP shock. Import prices of PCP firms follow exchange rate movements and thus instantaneously fall following the 1% appreciation. LCP firms do not fully pass-through the exchange rate change, however, and instead only reduce their price slowly and by less than 0.5%. This is because they are forward-looking and do not expect the exchange rate change to be permanent. Therefore, they increase their mark-up following the domestic appreciation (as shown in Figure 2). After a year, pass-through to import prices – as measured by the change in the level of import prices relative to the change in the level of the exchange rate – is approximately 90%, increasing to 100% within two years. The change in import prices slowly feeds through to the CPI according to the share of imports in the consumption basket.<sup>6</sup>

The behaviour of import prices following an exogenous exchange rate shock shows that mark-ups of exporters will move in the opposite direction of import prices, simply because of LCP and sticky prices. The extent to which the average mark-up increases (decreases) in the face of an appreciation (depreciation), however, will depend not only on the change in the exchange rate, but also on how the

<sup>6</sup> The monetary policymakers in the model are assumed to follow a flexible inflation targeting rule, so that they loosen monetary policy in response to the fall in the CPI. This is a simplification as it assumes that policymakers react in the same way to changes in the CPI whatever the origin or persistence of the change, ignoring other factors that are part of the monetary policy decision process.

shock causing that change affects the economy through other channels (especially expected demand and marginal costs as well as the future path of the exchange rate). Therefore, the final impact on prices will vary based on not just the magnitude of an exchange rate movement, but also the shock which caused this movement.

**Figure 2: The impact of an exogenous exchange rate change on selected variables**



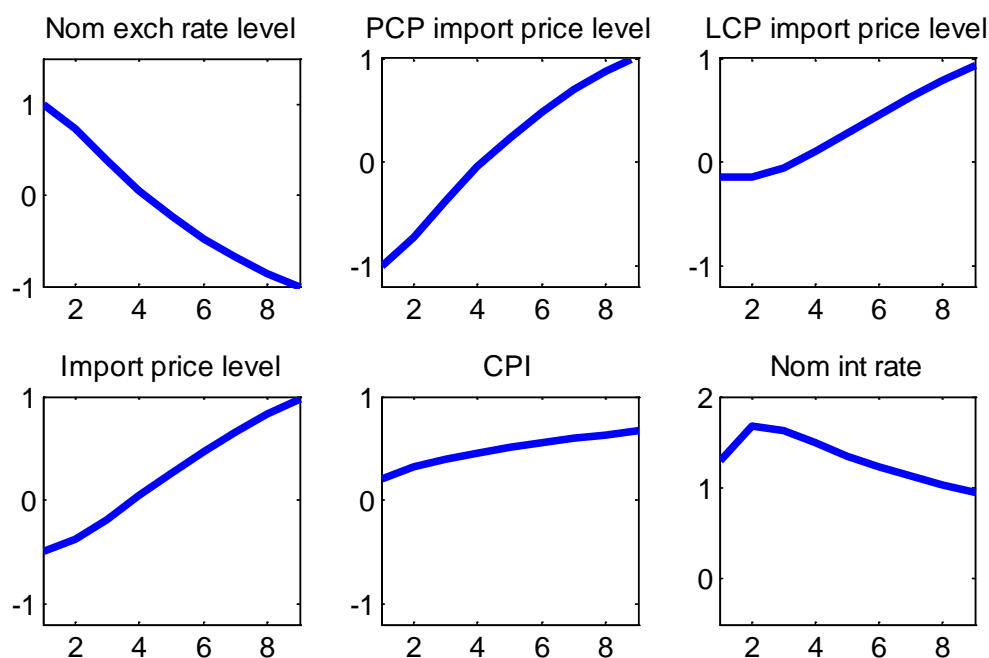
**Note:** This figure depicts the effects of an exogenous exchange rate shock causing the nominal exchange rate to appreciate by 1% in the first quarter on: the percent change in the level of the nominal exchange rate, in the import price level of PCP firms, in the import price level of LCP firms, in the overall import price level, in the CPI, and the percentage point change in the annualised nominal interest rate. The x-axis shows the quarters following the shock, which happens in quarter 1.

To clarify what mechanisms determine how pass-through might differ across shocks, we consider two additional examples of factors that could cause a similar 1% appreciation: stronger domestic demand and tighter monetary policy.

An appreciation caused by a positive *domestic demand shock* will increase the mark-up charged and the profits earned by foreign exporters who do not change their price, as explained above. The positive demand shock also increases domestic demand for imports, however, as well as domestic inflationary pressures. These effects will cause domestic competitors to increase prices. Moreover, the exchange rate impact of demand shocks is short-lived and therefore, in response to

higher domestic demand, foreign LCP exporters will face less pressure to reduce their prices.<sup>7</sup> As a result, import prices fall less than in the benchmark case of an exogenous exchange rate shock. These dynamics are shown in the simulations of the effects of a positive domestic demand shock in Figure 3.

**Figure 3: The impact of a demand shock on selected variables**



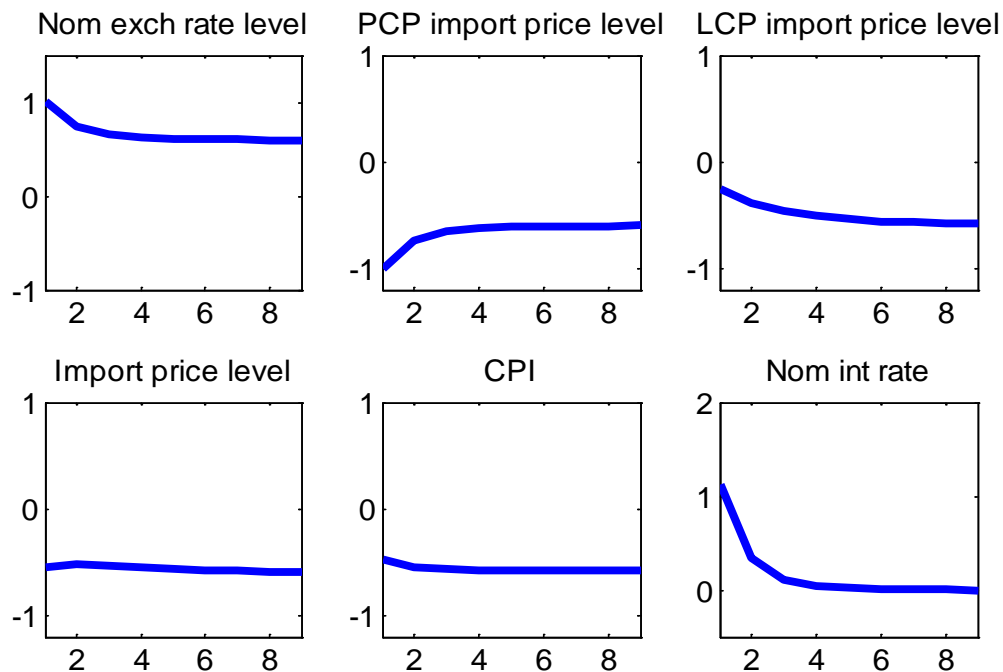
**Note:** This figure depicts the effects of a positive demand shock causing the nominal exchange rate to appreciate by 1% in the first quarter on: the percent change in the level of the nominal exchange rate, in the import price level of PCP firms, in the import price level of LCP firms, in the overall import price level, in the CPI, and the percentage point change in the annualised nominal interest rate. The x-axis shows the quarters following the shock, which happens in quarter 1.

Figure 1 compares the resulting mark-up from this scenario with the former example of an exogenous exchange rate shock. It shows that importers increase their mark-up more after the demand shock. As a result, pass-through is lower than after the UIP shock, with pass-through to import prices only around 85% after 4 quarters following the demand shock (relative to 90% in the previous scenario). Also, even though import prices fall as a consequence of the appreciation, the inflationary impact of the positive demand shock on the CPI more than outweighs the impact of lower import prices; the CPI rises despite the fall in import prices.

<sup>7</sup> The reason why the demand shock only leads to a temporary appreciation of the nominal exchange rate is our model assumption that PPP holds in the long run implying that the real exchange rate is stationary. The positive demand shock, while appreciating the domestic exchange rate in the short run, also increases domestic prices. Given that foreign prices are not affected by the domestic shock (and that is the case given our assumption that the domestic economy is small), the permanent increase in the domestic price level following the domestic demand shock must be mirrored by a depreciation of the nominal exchange rate in the long run to keep the real exchange rate stationary.

Finally, consider a *monetary policy shock* associated with an increase in the nominal interest rate which also leads to an appreciation of the nominal exchange rate of 1%. This appreciation reduces import prices, but the tighter monetary policy also reduces domestic demand and domestic inflationary pressures, as shown in Figure 4. Moreover, the resulting exchange rate appreciation is persistent. Exporters will therefore more fully incorporate the exchange rate move by reducing import prices, rather than increasing their margins. Indeed, Figure 2 shows that in this scenario, margins only increase by 0.2% after the appreciation—much less than in the previous two scenarios—and quickly fall back to zero. Exchange rate pass-through to the CPI will also be high, as the CPI falls more than it does in the face of the exogenous exchange rate shock and domestic demand shock.

**Figure 4: The impact of a monetary policy shock on selected variables**



**Note:** This figure depicts the effects of a monetary policy tightening causing the nominal exchange rate to appreciate by 1% in the first quarter on: the percent change in the level of the nominal exchange rate, in the import price level of PCP firms, in the import price level of LCP firms, in the overall import price level, in the CPI, and the percentage point change in the annualised nominal interest rate. The x-axis shows the quarters following the shock, which happens in quarter 1.

These illustrations show that even a standard model predicts that exporters vary their margins in response to different causes of an exchange rate movement and that pass-through is shock dependent. In the examples here, these margins depend not only on the exchange rate movement, but also on other factors, such as simultaneous and expected future changes in demand conditions related to the shock moving the exchange rate. For example, although the demand and monetary policy shocks moved the exchange rate in the same direction initially, they moved demand in opposite directions, thereby generating different implications for foreign exporters' mark-ups and pass-through. Exchange rate pass-through also varies across shocks because the persistence of the effect

on the exchange rate differs across shocks. Varying the persistence of shocks within plausible ranges, however, while important in affecting exchange rate pass-through following each of the shocks, does not affect our conclusions that demand shocks tend to be associated with lower degrees of exchange rate pass-through while monetary policy shocks tend to be associated with higher degrees. In our framework, exchange rate pass-through could also vary across shocks because the shocks have different effects on exporters' future marginal costs (especially if the shocks are global). In any of these cases, theory clearly predicts that pass-through differs across shocks. We use these insights in our empirical estimation of pass-through which explicitly incorporates pass-through as shock-dependent.

### DSGE model equilibrium equations

The equilibrium is a set of stationary processes

$$\left\{ \begin{array}{l} Y_{H,t}, Y_{F,t}, C_t, C_t^*, L_t, L_t^*, C_{H,t}, C_{H,t}^*, C_{F,t}, C_{F,t}^*, Q_t, \left(\frac{B_{F,t}}{P_t}\right), i_t, i_t^* \\ \left(\frac{P_{H,t}}{P_t}\right), \left(\frac{P_{F,t}}{P_t}\right), \left(\frac{P_{H,t}^{PCP}}{P_{H,t}}\right), \left(\frac{P_{H,t}^{LCP}}{P_{H,t}}\right), \left(\frac{P_{F,t}^{LCP}}{P_{F,t}}\right), \left(\frac{P_{F,t}^*}{P_t^*}\right), \left(\frac{P_{H,t}^*}{P_t^*}\right), \left(\frac{P_{F,t}^{PCP^*}}{P_{F,t}^*}\right), \left(\frac{P_{H,t}^{LCP^*}}{P_{H,t}^*}\right), \left(\frac{P_{F,t}^{LCP^*}}{P_{F,t}^*}\right), \\ \pi_{H,t}, \pi_{H,t}^{PCP}, \pi_{H,t}^{LCP}, \pi_{H,t}^{LCP^*}, \pi_{F,t}^*, \pi_{F,t}^{PCP^*}, \pi_{F,t}^{LCP^*}, \pi_{F,t}^{LCP}, \left(\frac{W_t}{P_t}\right), \left(\frac{W_t^*}{P_t^*}\right), \\ x_{1,t}^{PCP}, x_{2,t}^{PCP}, x_{1,t}^{LCP}, x_{1,t}^{LCP^*}, x_{2,t}^{LCP}, x_{2,t}^{LCP^*}, x_{F,1,t}^{PCP}, x_{F,2,t}^{PCP}, x_{F,1,t}^{LCP}, x_{F,1,t}^{LCP^*}, x_{F,2,t}^{LCP}, x_{F,2,t}^{LCP^*} \end{array} \right\} \text{ for } t \geq 0$$

which satisfy the 46 equilibrium equations described in this appendix (denoted 1A-46A) given the shock processes for  $\{\gamma_t^C, \gamma_t^{C^*}, \gamma_t^I, \gamma_t^{I^*}, \gamma_t^S\}_{t=0}^\infty$  and the initial conditions consisting of the variables above for  $t < 0$ .

### Price equations

We rewrite the pricing equations in a recursive form by engaging in the following transformations.

Focusing first on the PCP firms' maximisation problem, we rewrite the following pricing equation

$$\frac{p_t^{PCP}(h)}{P_{H,t}^{PCP}} = \frac{\theta_H}{(\theta_H - 1)(1 - \tau_H)} \frac{E_t \sum_{s=0}^\infty (\beta \alpha_H)^s P_{t+s}^{-1} U_{C,t+s} W_{t+s} y_{t,t+s}^{PCP}(h)}{E_t \sum_{s=0}^\infty (\beta \alpha_H)^s P_{H,t}^{PCP} P_{t+s}^{-1} U_{C,t+s} y_{t,t+s}^{PCP}(h)}$$

as

$$\frac{p_t^{PCP}(h)}{P_{H,t}^{PCP}} = \frac{x_{1,t}^{PCP}}{x_{2,t}^{PCP}}$$

where  $x_{1,t}^{PCP} \equiv \frac{\theta_H}{(\theta_H - 1)} E_t \sum_{s=0}^\infty (\beta \alpha_H)^s P_{t+s}^{-1} U_{C,t+s} W_{t+s} y_{t,t+s}^{PCP}(h)$

$$= \frac{\theta_H}{(\theta_H - 1)} C_t^{-\sigma} \frac{W_t}{P_t} p_{H,t}^{PCP - \theta_H} \left\{ C_{H,t} + \frac{1-n}{n} p_{H,t}^{-\theta_H} Q_t^{\theta_H} p_{H,t}^* \theta_H C_{H,t}^* \right\} + (\beta \alpha_H) \pi_{H,t}^{PCP} \theta_H x_{1,t+1}^{PCP} \quad (1A)$$

and  $x_{2,t}^{PCP} \equiv (1 - \tau_H) E_t \sum_{s=0}^{\infty} (\beta \alpha_H)^s P_{H,t}^{PCP} P_{t+s}^{-1} U_{C,t+s} y_{t,t+s}^{PCP}(h) = (1 - \tau_H) C_t^{-\sigma} p_{H,t} p_{H,t}^{PCP 1 - \theta_H} \left\{ C_{H,t} + \frac{1-n}{n} p_{H,t}^{-\theta_H} Q_t^{\theta_H} p_{H,t}^* \theta_H C_{H,t}^* \right\} + (\beta \alpha_H) \pi_{H,t}^{PCP \theta_H - 1} x_{2,t+1}^{PCP}$  (2A)

where  $p_{H,t} \equiv \left( \frac{P_{H,t}}{P_t} \right)$ ,  $p_{H,t}^{PCP} \equiv \left( \frac{P_{H,t}^{PCP}}{P_{H,t}} \right)$ , and  $p_{H,t}^* \equiv \left( \frac{P_{H,t}^*}{P_t^*} \right)$ .

Then, we note that

$$P_{H,t}^{PCP} = \left[ \alpha_H^k P_{H,t-1}^{PCP 1 - \theta_H} + (1 - \alpha_H^k) p_t^{PCP}(h)^{1 - \theta_H} \right]^{\frac{1}{1 - \theta_H}}$$

can be rewritten as

$$1 = \alpha_H^k \left( \frac{P_{H,t-1}^{PCP}}{P_{H,t}^{PCP}} \right)^{1 - \theta_H} + (1 - \alpha_H^k) \left( \frac{p_t^{PCP}(h)}{P_{H,t}^{PCP}} \right)^{1 - \theta_H}$$

So that

$$\frac{p_t^{PCP}(h)}{P_{H,t}^{PCP}} = \left[ \frac{1 - \alpha_H^k (\pi_{H,t}^{PCP})^{\theta_H - 1}}{(1 - \alpha_H^k)} \right]^{\frac{1}{1 - \theta_H}}$$

Implying that

$$\left[ \frac{1 - \alpha_H^k (\pi_{H,t}^{PCP})^{\theta_H - 1}}{(1 - \alpha_H^k)} \right]^{\frac{1}{1 - \theta_H}} = \frac{x_{1,t}^{PCP}}{x_{2,t}^{PCP}} \quad (3A)$$

Now, turning to LCP firms, we can rewrite the pricing equation for domestic prices as before to obtain:

$$\left[ \frac{1 - \alpha_H^k (\pi_{H,t}^{LCP})^{\theta_H - 1}}{(1 - \alpha_H^k)} \right]^{\frac{1}{1 - \theta_H}} = \frac{x_{1,t}^{LCP}}{x_{2,t}^{LCP}} \quad (4A)$$

where  $x_{1,t}^{LCP} \equiv \frac{\theta_H}{(\theta_H - 1)} E_t \sum_{s=0}^{\infty} (\beta \alpha_H)^s P_{t+s}^{-1} U_{C,t+s} W_{t+s} y_{t,t+s}^{LCP}(h)$

$$= \frac{\theta_H}{(\theta_H - 1)} C_t^{-\sigma} \frac{W_t}{P_t} p_{H,t}^{LCP - \theta_H} C_{H,t} + (\beta \alpha_H) \pi_{H,t}^{LCP \theta_H} x_{1,t+1}^{LCP} \quad (5A)$$

and  $x_{2,t}^{LCP} \equiv (1 - \tau_H) E_t \sum_{s=0}^{\infty} (\beta \alpha_H)^s P_{H,t}^{LCP} P_{t+s}^{-1} U_{C,t+s} y_{t,t+s}^{LCP}(h)$

$$= (1 - \tau_H) C_t^{-\sigma} p_{H,t} p_{H,t}^{LCP 1 - \theta_H} C_{H,t} + (\beta \alpha_H) \pi_{H,t}^{LCP \theta_H - 1} x_{2,t+1}^{LCP} \quad (6A)$$

The pricing equation for exported goods amounts to:

$$\left[ \frac{1 - \alpha_H^k (\pi_{H,t}^{LCP*})^{\theta_H - 1}}{(1 - \alpha_H^k)} \right]^{\frac{1}{1 - \theta_H}} = \frac{x_{1,t}^{LCP*}}{x_{2,t}^{LCP*}} \quad (7A)$$

$$\begin{aligned}
\text{where } x_{1,t}^{LCP*} &\equiv \frac{\theta_H}{(\theta_H-1)} E_t \sum_{s=0}^{\infty} (\beta \alpha_H)^s P_{t+s}^{-1} U_{C,t+s} W_{t+s} y_{t,t+s}^{LCP*}(h) \\
&= \frac{\theta_H}{(\theta_H-1)} C_t^{-\sigma} \frac{W_t}{P_t} p_{H,t}^{LCP*-\theta_H} C_{H,t}^* + (\beta \alpha_H) \pi_{H,t}^{LCP* \theta_H} x_{1,t+1}^{LCP*}
\end{aligned} \tag{8A}$$

$$\begin{aligned}
\text{and } x_{2,t}^{LCP*} &\equiv (1 - \tau_H) E_t \sum_{s=0}^{\infty} (\beta \alpha_H)^s S_{t+s} P_{H,t}^{LCP*} P_{t+s}^{-1} U_{C,t+s} y_{t,t+s}^{LCP*}(h) \\
&= (1 - \tau_H) C_t^{-\sigma} p_{H,t}^* Q_t p_{H,t}^{LCP*1-\theta_H} C_{H,t}^* + (\beta \alpha_H) \pi_{H,t}^{LCP* \theta_H-1} x_{2,t+1}^{LCP*}
\end{aligned} \tag{9A}$$

$$\text{where } p_{H,t}^{LCP*} = \frac{P_{H,t}^{LCP*}}{P_{H,t}^*}.$$

Similar equations hold for the Foreign LCP and PCP firms, so that there are in total 18 pricing equations.

Consumption demand:

$$C_{H,t} = a_H \left( \frac{P_{H,t}}{P_t} \right)^{-\varphi_H} C_t, \tag{19A}$$

$$C_{F,t} = (1 - a_H) \left( \frac{P_{F,t}}{P_t} \right)^{-\varphi_H} C_t, \tag{20A}$$

$$C_{H,t}^* = (1 - a_F) \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-\varphi_F} C_t^*, \tag{21A}$$

$$C_{F,t}^* = a_F \left( \frac{P_{F,t}^*}{P_t^*} \right)^{-\varphi_F} C_t^* \tag{22A}$$

Labour supply:

$$\frac{L_t^\eta}{\gamma_t^C C_t^{-\sigma}} = \frac{W_t}{P_t}, \tag{23A}$$

$$\frac{L_t^{*\eta}}{\gamma_t^{C*} C_t^{*-\sigma}} = \frac{W_t^*}{P_t^*}, \tag{24A}$$

Consumption Euler equations:

$$\beta E_t \frac{\gamma_{t+1}^C C_{t+1}^{-\sigma} (1+i_t)}{\gamma_t^C C_t^{-\sigma} \pi_{t+1}} = 1 \tag{25A}$$

$$\beta E_t \frac{\gamma_{t+1}^C C_{t+1}^{-\sigma} (1+i_t^*) Q_{t+1}}{\gamma_t^C C_t^{-\sigma} \pi_{t+1}^* Q_t} = \frac{1}{\Phi \left( \frac{S_t B_{F,t}}{P_t} \right)} \gamma_t^S \tag{26A}$$

$$\beta E_t \frac{\gamma_{t+1}^{C*} C_{t+1}^{*-\sigma} (1+i_t^*)}{\gamma_t^{C*} C_t^{*-\sigma} \pi_{t+1}^*} = 1 \tag{27A}$$



where  $\pi_t \equiv \frac{P_t}{P_{t-1}} = \frac{p_{H,t-1}}{p_{H,t}} \pi_{H,t}$  denotes CPI inflation in the Home country and  $\pi_t^* \equiv \frac{p_{F,t-1}^*}{p_{F,t}^*} \pi_{F,t}^*$  denotes CPI inflation in the Foreign economy.

Price indices:

$$1 = a_H \left( \frac{P_{H,t}}{P_t} \right)^{1-\varphi_H} + (1 - a_H) \left( \frac{P_{F,t}}{P_t} \right)^{1-\varphi_F} \quad (28A)$$

$$1 = a_F \left( \frac{P_{F,t}^*}{P_t^*} \right)^{1-\varphi_F} + (1 - a_F) \left( \frac{P_{H,t}^*}{P_t^*} \right)^{1-\varphi_H} \quad (29A)$$

$$1 = \gamma_H^{PCP} \left( \frac{P_{H,t}^{PCP}}{P_{H,t}} \right)^{1-\theta_H} + (1 - \gamma_H^{PCP}) \left( \frac{P_{H,t}^{LCP}}{P_{H,t}} \right)^{1-\theta_H} \quad (30A)$$

$$1 = \gamma_H^{PCP} \left( \frac{P_{H,t}^{PCP}}{P_{H,t}} \frac{P_{H,t}}{P_t} \frac{P_t}{s_t P_t^*} \frac{P_t^*}{P_{H,t}^*} \right)^{1-\theta_H} + (1 - \gamma_H^{PCP}) \left( \frac{P_{H,t}^{LCP^*}}{P_{H,t}^*} \right)^{1-\theta_H} \quad (31A)$$

$$1 = \gamma_F^{PCP} \left( \frac{P_{F,t}^{PCP^*}}{P_{F,t}^*} \right)^{1-\theta_F} + (1 - \gamma_F^{PCP}) \left( \frac{P_{F,t}^{LCP^*}}{P_{F,t}^*} \right)^{1-\theta_F} \quad (32A)$$

$$1 = \gamma_F^{PCP} \left( \frac{P_{F,t}^{PCP^*}}{P_{F,t}^*} \frac{P_{F,t}^*}{P_t^*} \frac{s_t P_t^*}{P_t} \frac{P_t}{P_{F,t}} \right)^{1-\theta_F} + (1 - \gamma_F^{PCP}) \left( \frac{P_{F,t}^{LCP}}{P_{F,t}} \right)^{1-\theta_F} \quad (33A)$$

Definition of inflation rates:

$$\pi_{H,t}^{PCP} = \frac{P_{H,t}^{PCP}}{P_{H,t}} \frac{P_{H,t-1}}{P_{H,t-1}^{PCP}} \pi_{H,t} \quad (34A)$$

$$\pi_{F,t}^{PCP^*} = \frac{P_{F,t}^{PCP^*}}{P_{F,t}^*} \frac{P_{F,t-1}^*}{P_{F,t-1}^{PCP^*}} \pi_{F,t}^* \quad (35A)$$

$$\pi_{H,t}^{LCP} = \frac{P_{H,t}^{LCP}}{P_{H,t}} \frac{P_{H,t-1}}{P_{H,t-1}^{LCP}} \pi_{H,t} \quad (36A)$$

$$\pi_{H,t}^{LCP^*} = \frac{P_{H,t}^{LCP^*}}{P_{H,t}^*} \frac{P_{H,t-1}^*}{P_{H,t-1}^{LCP^*}} \frac{P_{H,t}}{P_t} \frac{P_{t-1}}{P_{H,t-1}^*} \frac{P_{F,t-1}^*}{P_{t-1}^*} \frac{P_t}{P_{F,t}^*} \pi_{F,t}^* \quad (37A)$$

$$\pi_{F,t}^{LCP^*} = \frac{P_{F,t}^{LCP^*}}{P_{F,t}^*} \frac{P_{F,t}^*}{P_{F,t-1}^{LCP^*}} \pi_{F,t}^* \quad (38A)$$

$$\pi_{F,t}^{LCP} = \frac{P_{F,t}^{LCP}}{P_{F,t}} \frac{P_{F,t-1}}{P_{F,t-1}^{LCP}} \frac{P_{F,t}}{P_t} \frac{P_{t-1}}{P_{F,t-1}} \frac{P_t}{P_{H,t}} \frac{P_{H,t-1}}{P_{t-1}} \pi_{H,t} \quad (39A)$$

Production functions:<sup>8</sup>

$$Y_{H,t} = L_t \quad (40A)$$

$$Y_{F,t}^* = L_t^* \quad (41A)$$

Resource constraint:

The resource constraint is  $C_t + \frac{s_t B_{F,t}}{P_t(1+i_t^*)\Phi\left(\frac{s_t B_{F,t}}{P_t}\right)} = Y_t + \frac{s_t B_{F,t-1}}{P_t}$  which we can rewrite as:

$$C_t + \frac{Q_t(B_{F,t}/P_t^*)}{(1+i_t^*)\Phi(Q_t(B_{F,t}/P_t^*))} = Y_t + \frac{Q_t(B_{F,t-1}/P_{t-1}^*)}{\pi_t^*} \quad (42A)$$

Monetary policy rules:

$$\log\left(\frac{i_t}{\bar{i}}\right) = \alpha_H^R \log\left(\frac{i_{t-1}}{\bar{i}}\right) + \alpha_H^\pi \log\left(\frac{\pi_t}{\bar{\pi}}\right) + \gamma_t^I \quad (43A)$$

$$\log\left(\frac{i_t^*}{\bar{i}}\right) = \alpha_F^R \log\left(\frac{i_{t-1}^*}{\bar{i}}\right) + \alpha_F^\pi \log\left(\frac{\pi_t^*}{\bar{\pi}}\right) + \gamma_t^{I*} \quad (44A)$$

Goods market equilibrium:

$$Y_{H,t} = a_H \left(\frac{P_{H,t}}{P_t}\right)^{-\varphi} C_t + \frac{1-n}{n} (1 - a_F) \left(\frac{P_{H,t}}{P_t}\right)^{-\varphi} Q_t^\varphi C_t^* \quad (45A)$$

$$Y_{F,t} = \frac{n}{1-n} (1 - a_H) \left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\varphi} Q_t^{-\varphi} C_t + a_F \left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\varphi} C_t^* \quad (46A)$$

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<sup>8</sup> For simplicity, we here do not account for the impact of price dispersion on output. Given that price dispersion does not play a role in first order approximations of the equilibrium, this does not have any consequences for our results.